Name: Solutions

Venn Diagrams

Showing Probabilities

Date:

Time:

Total marks available:

Total marks achieved: _____

Questions

Q1.

(a) State in words the relationship between two events R and S when $P(R \cap S) = 0$

mutually exclusive

(1)

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$

$\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

$P(A) \times P(B) = P(A \cap B)$

Find

Let $P(B) = x$

(b) $P(B)$

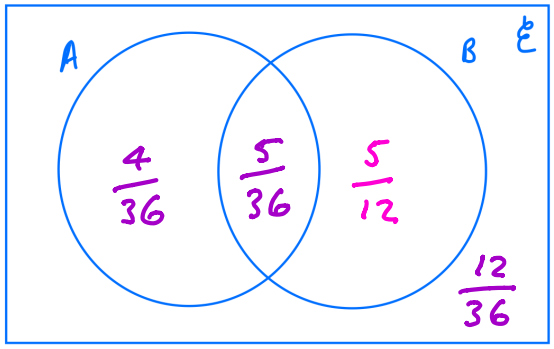
$\frac{1}{4} \times x = x - \frac{5}{12}$

$3x = 12x - 5$

$5 = 9x$

$x = \frac{5}{9}$

$= \frac{5}{9}$



(4)

(c) $P(A' \cap B)$

$= \frac{5}{12}$

(2)

(d) $P(B' | A)$

$= \frac{4}{9}$

(2)

(Total 9 marks)

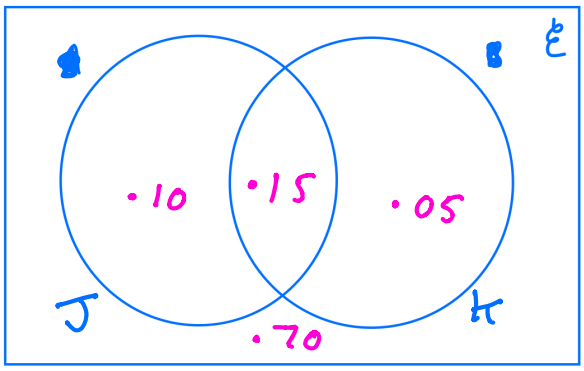
Q2.

Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school

K = the event that Jake is late for school

$P(J) = 0.25$, $P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$



On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

0·3

(1)

(b) Kamil is late for school.

0·2

(2)

Given that Jake is late for school,

(c) find the probability that Kamil is late.

$\frac{0 \cdot 15}{0 \cdot 25} = \frac{3}{5}$

(3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

$P(J) \times P(K) = 0.25 \times 0.2 = 0.05 \neq 0.15 = P(J \cap K)$

so not independent

(2)

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

Suspicion seems to be correct

(1)

(Total 9 marks)

Q3.

(a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

(i) A and B are mutually exclusive, $P(A \cup B) = a + b$

(ii) A and B are independent. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= a + b - ab$

(2)

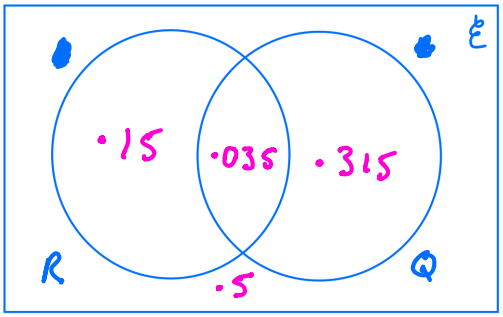
Two events R and Q are such that

$P(R \cap Q') = 0.15$, $P(Q) = 0.35$ and $P(R|Q) = 0.1$

$P(R|Q) = \frac{P(R \cap Q)}{P(Q)}$ $\quad 0.1 = \frac{P(R \cap Q)}{0.35}$

Find the value of

(b) $P(R \cup Q)$, $= 0.5$



(1)

(c) $P(R \cap Q)$, $= 0.035$

(2)

(d) $P(R)$. $= 0.185$

(2)

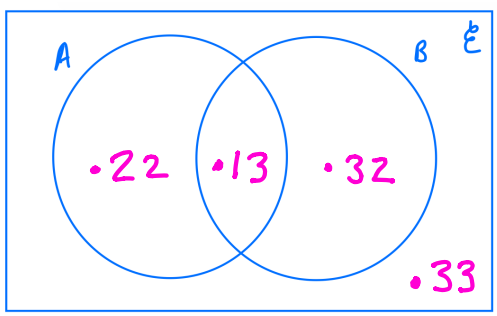
(Total 7 marks)

Q4.

Given that

$P(A) = 0.35$, $P(B) = 0.45$ and $P(A \cap B) = 0.13$

find $(A \cup B)$ $= 0.67$



(2)

(b) $P(A' \mid B')$ $= \frac{.33}{.55} = \frac{3}{5}$

(2)

The event C has P(C) = 0.20

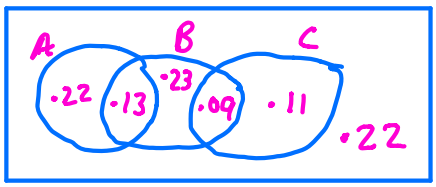
The events A and C are mutually exclusive and the events B and C are independent.

(c) Find P($B \cap C$) $= P(B) \times P(C)$ $= .45 \times 0.2 = 0.09$

(2)

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region.

(4)



Tom invites Avisha to play a game with these dice.

(e) Find P([$B \cup C$]′) $= 0.44$

(2)

(Total 12 marks)

Q5.

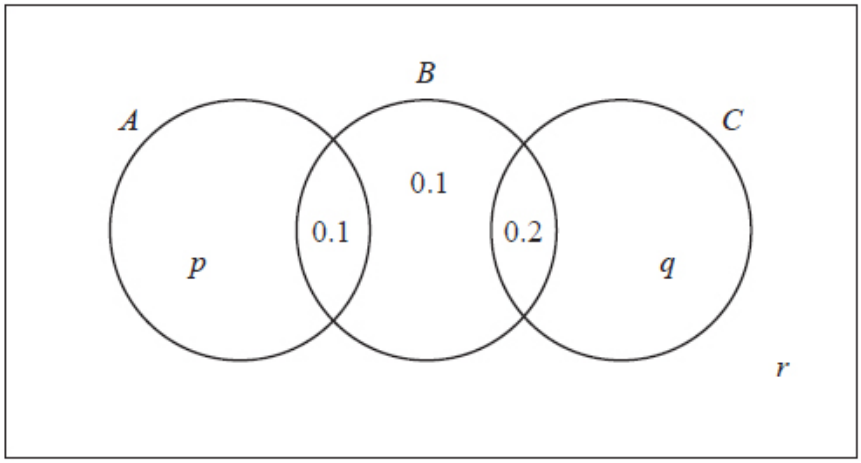


Figure 1

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

$P(A) \times P(B) = P(A \cap B)$

$(p + 0.1) \times 0.4 = 0.1$

$p + 0.1 = \frac{0.1}{0.4} = 0.25$

(a) Find the value of p. $p = 0.15$

(3)

Given that P($B \mid C$) = $\frac{5}{11}$ $= \frac{P(B \cap C)}{P(C)} = \frac{0.2}{q + 0.2}$

$5q + 1 = 2.2$

$5q = 1.2$

$q = 0.24$

(b) find the value of q and the value of r.

(4)

$r = 1 - (0.4 + 0.15 + 0.24) = 0.21$ $r = 0.21$

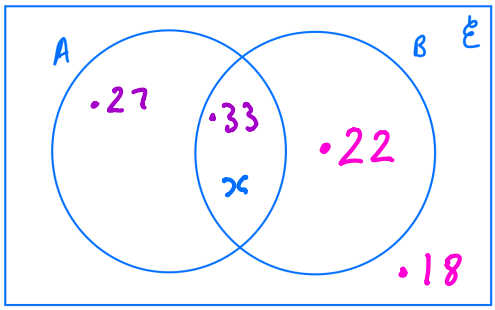
(c) Find P($A \cup C \mid B$). $= \frac{0.1 + 0.2}{0.4} = \frac{3}{4} = 0.75$

(2)

(Total 9 marks)

Q6.

For the events A and B,



$P(A' \cap B) = 0.22$ and $P(A' \cap B') = 0.18$

(a) Find P(A). $P(A) = 1 - 0.18 - 0.22 = 0.6$

(1)

(b) Find P($A \cup B$). $P(A \cup B) = 1 - 0.18 = 0.82$

(1)

Given that $P(A \mid B) = 0.6$ $= \frac{P(A \cap B)}{P(B)} = \frac{x}{x + 0.22}$

(c) find P($A \cap B$).

$0.6x + 0.132 = x$

$0.132 = 0.4x$ $\quad x = \frac{0.132}{0.4} = 0.33$

(3)

(d) Determine whether or not A and B are independent.

$P(A \cap B) = 0.33$

$P(A) \times P(B)$ $\qquad P(A \cap B)$

0.6×0.55

$0.33 \quad = \quad 0.33$

$\therefore$ independent

(2)

(Total 7 marks)

Q7.

A and B are two events such that

$P(B) = \frac{1}{2}$ $\quad P(A \mid B) = \frac{2}{5}$ $\quad P(A \cup B) = \frac{13}{20}$

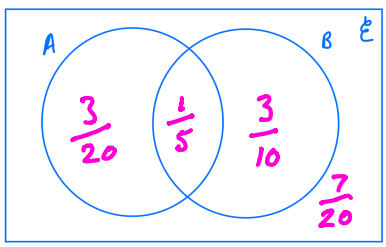
(a) Find P($A \cap B$). $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $\quad P(A \cap B) = P(A \mid B) \times P(B)$

$= \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$

$= \frac{1}{5}$

(2)

(b) Draw a Venn diagram to show the events A, B and all the associated probabilities.



(3)

Find

(c) P(A) $= \frac{3}{20} + \frac{1}{5} = \frac{7}{20}$

(1)

(d) P($B \mid A$) $= \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{1}{5} \times \frac{20}{7} = \frac{4}{7}$

(2)

(e) P($A' \cap B$)

$= \frac{3}{10}$

(1)

(Total 9 marks)

Q8.

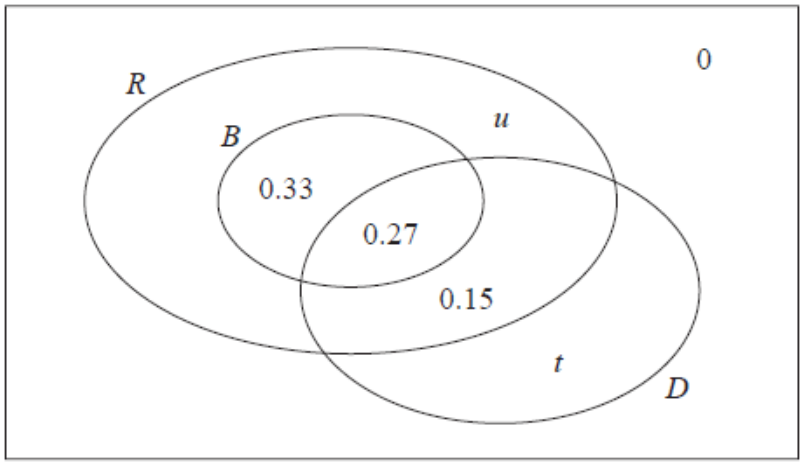
The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room

B is the event that a customer books breakfast

D is the event that a customer books dinner

u and t are probabilities.



(a) Write down the probability that a customer books breakfast but does not book a room.

0

(1)

Given that the events B and D are independent

(b) find the value of t

$P(B) \times P(D) = P(B \cap D)$

$0.6 \times (0.42 + t) = 0.27$

$0.42 + t = \frac{0.27}{0.6} = 0.45$

$t = 0.03$

(4)

(c) hence find the value of u

$u = 1 - (0.33 + 0.27 + 0.15 + 0.03) = 0.22$

$u = 0.22$

(2)

(d) Find

(i) $P(D|R \cap B)$ $= \frac{.27}{.60} = 0.45$

(ii) $P(D|R \cap B')$ $= \frac{.15}{.37} = \frac{15}{37}$ **(4)**

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast

37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.

$40 \times 0.45 + 37 \times \frac{15}{37}$ **(2)**

$= 18 + 15$

$= 33$

(Total for question = 13 marks)