9. A geometric series has first term a and common ratio r.
The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.
(4)

(b) Find the two possible values of r.
(2)

(c) Find the corresponding two possible values of a.
(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$
(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.
(2)

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a) 2nd term $ar = 4 \Rightarrow a = \frac{4}{r}$

$S_\infty \quad \frac{a}{1-r} = 25$

$S_\infty \quad \frac{\frac{4}{r}}{1-r} = 25$

$\frac{4}{r} = 25(1-r)$

$4 = 25r(1-r)$

$4 = 25r - 25r^2$

$\underline{25r^2 - 25r + 4 = 0}$

b) $r = \frac{4}{5}$ $\quad r = \frac{1}{5}$ by calc

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c) $a = \frac{4}{r}$ $\quad \frac{4}{\frac{4}{5}} = 4 \times \frac{5}{4} = 5$

$$a = \frac{4}{\frac{1}{5}} = \frac{4 \times 5}{1} = 20$$

$$\begin{cases} a = 20 \\ r = \frac{1}{5} \end{cases} \qquad \begin{cases} a = 5 \\ r = \frac{4}{5} \end{cases}$$

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d)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$a = 20$, $r = \frac{1}{5}$

$$S_n = \frac{20(1-r^n)}{1-\frac{1}{5}} = \frac{20(1-r^n)}{\frac{4}{5}}$$

$$= 25(1-r^n)$$

$a = 5$, $r = \frac{4}{5}$

$$S_n = \frac{5(1-r^n)}{1-\frac{4}{5}} = \frac{5(1-r^n)}{\frac{1}{5}}$$

$$= 25(1-r^n)$$

$$\therefore \quad S_n = 25(1-r^n)$$

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e) $r = \frac{4}{5}$, $a = 5$

$$S_n = 25(1-r^n)$$

$$S_n > 24$$

$$\therefore \quad 25(1-r^n) > 24$$

$$1 - r^n > \frac{24}{25}$$

$$1 - \frac{24}{25} > r^n$$

$$\frac{1}{25} > r^n$$

$$\ln 0.04 > \ln 0.8^n$$

$\ln 0.04 > n \ln 0.8$

$\frac{\ln 0.04}{\ln 0.8} < n$

$14.4 < n$

Smallest n $\quad n = 15$

1. The first three terms of a geometric series are

18, 12 and p

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series, **(1)**

(b) the value of p, **(1)**

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places. **(2)**

a) 1st $\quad a = 18 \quad$ 1

2nd $\quad ar = 12 \quad$ 2

3rd $\quad ar^2 = p \quad$ 3

2 ÷ 1 $\quad \frac{ar}{a} = r = \frac{12}{18} = \frac{2}{3} \qquad r = \frac{2}{3}$

b) 3 ÷ 2 $\quad \frac{p}{12} = \frac{2}{3} \qquad 3p = 24 \qquad p = 8$

c) $S_n = \frac{a(1-r^n)}{1-r} \qquad S_{15} = \frac{18(1-(\frac{2}{3})^{15})}{(1-\frac{2}{3})}$

$= 53.877$

6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

a) $a = 20 \qquad r = \frac{7}{8}$

$$S_\infty = \frac{a}{1-r} = \frac{20}{1-\frac{7}{8}} = \frac{20}{\frac{1}{8}} = 160$$

b) $$S_n = \frac{a(1-r^n)}{1-r} \qquad S_{12} = \frac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}} = 127.8$$

c) $$S_\infty - S_N < 0.5$$

$$160 - 0.5 < S_N$$

$$159.5 < \frac{a(1-r^N)}{1-r}$$

$$159.5 < \frac{20(1-r^N)}{1-\frac{7}{8}}$$

$$159.5 < 160(1-r^N)$$

$$\frac{159.5}{160} < 1 - r^N$$

$$r^N < 1 - \frac{159.5}{160}$$

$$r^N < \frac{0.5}{160}$$

$$r^N < \frac{1}{320}$$

$$\ln r^N < \ln\left(\frac{1}{320}\right)$$

$$N \ln r < \ln\left(\frac{1}{320}\right)$$

$$N > \frac{\ln\left(\frac{1}{320}\right)}{\ln\left(\frac{7}{8}\right)}$$

$$N > 43.198$$

Smallest N $\quad N = 44$