10. The first term of a geometric series is 120. The sum to infinity of the series is 480.

(a) Show that the common ratio, r, is $\frac{3}{4}$.

(3)

(b) Find, to 2 decimal places, the difference between the 5th and 6th term.

(2)

(c) Calculate the sum of the first 7 terms.

(2)

The sum of the first n terms of the series is greater than 300.

(d) Calculate the smallest possible value of n.

(4)

(Total 11 marks)

a) $S_\infty = \frac{a}{1-r}$

$480 = \frac{120}{1-r}$

$480(1-r) = 120$

$1-r = \frac{120}{480}$

$1-r = \frac{1}{4}$

$1-\frac{1}{4} = r$

$r = \frac{3}{4}$

b) n^{th} term $= ar^{n-1}$

5^{th} term $= ar^4 = 120 \times 0.75^4 = 37.96875$

6^{th} term $= ar^5 = 120 \times 0.75^5 = 28.4765625$

$37.96875 - 28.4765625 = 9.49$

c) $S_n = \frac{a(1-r^n)}{1-r}$

$S_7 = \frac{120(1-0.75^7)}{1-0.75}$

$S_7 = 415.93$

d)

$$S_n = \frac{a(1-r^n)}{1-r} > 300$$

$$\frac{120(1-0.75^n)}{\frac{1}{4}} > 300$$

$$120(1-0.75^n) > 75$$

$$1-0.75^n > \frac{75}{120}$$

$$1-\frac{75}{120} > 0.75^n$$

$$\frac{45}{120} > 0.75^n$$

$$\log\left(\frac{45}{120}\right) > \log 0.75^n$$

$$\log\left(\frac{45}{120}\right) > n \log 0.75$$

$$\frac{\log\left(\frac{45}{120}\right)}{\log 0.75} < n$$

$$3.409 < n$$

Smallest n $\quad n = 4$