6 A sequence is given by

$$a_1 = 4,$$

$$a_{r+1} = a_r + 3.$$

Write down the first 4 terms of this sequence.

Find the sum of the first 100 terms of the sequence. [5]

4752 January 2005

$a_1 = 4$

$a_2 = 4 + 3 = 7$

$a_3 = 7 + 3 = 10$

$a_4 = 10 + 3 = 13$

A.P.

$a = 4$

$d = 3$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{100} = \frac{100}{2}\left(2(4) + 99 \times 3\right) = 15,250$

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2 The nth term of an arithmetic progression is $6 + 5n$. Find the sum of the first 20 terms. [4]

1st $6 + 5(1) = 11$

2nd $6 + 5(2) = 16$

3rd $6 + 5(3) = 21$

A.P.

$a = 11$

$d = 5$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{20} = \frac{20}{2}\left(2(11) + 19 \times 5\right) = 1170$

---

12 **(i)** Granny gives Simon £5 on his 1st birthday. On each successive birthday, she gives him £2 more than she did the previous year.

(*A*) How much does she give him on his 10th birthday? [2]

(*B*) How old is he when she gives him £51? [2]

(*C*) How much has she given him **in total** when he has had his 20th birthday present? [2]

(ii) Grandpa gives Simon £5 on his 1st birthday and increases the amount by 10% each year.

(*A*) How much does he give Simon on his 10th birthday? [2]

(*B*) Simon first gets a present of over £50 from Grandpa on his *n*th birthday. Show that

$$n > \frac{1}{\log_{10} 1.1} + 1 .$$

Find the value of *n*. [5]

i) A.P $a = 5$, $d = 2$

A) $10^{th} = a + 9d = 5 + 9 \times 2 = £23$

B) n^{th} term $= a + (n-1)d = 51$

$5 + (n-1) \times 2 = 51$

$2(n-1) = 51 - 5$

$2(n-1) = 46$

$n - 1 = \frac{46}{2}$

$n - 1 = 23$

$\underline{n = 24}$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2(5) + 19 \times 2) = £480$

6 A sequence is given by the following.

$$u_1 = 3$$
$$u_{n+1} = u_n + 5$$

(i) Write down the first 4 terms of this sequence. [1]

(ii) Find the sum of the 51st to the 100th terms, inclusive, of the sequence. [4]

i)

$U_1 = 3$

$U_2 = 3 + 5 = 8$

$U_3 = 8 + 5 = 13$

$U_4 = 13 + 5 = 18$

A.P

$a = 3$

$d = 5$

ii)

Sum of 51st to 100th

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= S_{100} - S_{50}$

$= \frac{100}{2}\left(2(3) + 99(5)\right) - \frac{50}{2}\left(2(3) + 49 \times 5\right)$

$= 25050 - 6275$

$= 18775$

Exercise

8 The 7th term of an arithmetic progression is 6. The sum of the first 10 terms of the progression is 30.

Find the 5th term of the progression. [5]

7th $a + 6d = 6$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{10} = \frac{10}{2}\left(2a + 9d\right) = 30$

$10a + 45d = 30$

$$a + 6d = 6 \quad (1)$$
$$10a + 45d = 30 \quad (2)$$

from (1) $a = 6 - 6d$

sub in (2) $10(6 - 6d) + 45d = 30$

$$60 - 60d + 45d = 30$$
$$-15d = -30$$
$$\underline{d = 2}$$

$$a = 6 - 6(2)$$
$$\underline{a = -6}$$

5th term $= a + 4d$

$= -6 + 4(2)$

5th term $= 2$

Exercise 3A

9) AP

1st	2nd	3rd
-8	k^2	$17k$

$$d = k^2 - -8 = k^2 + 8$$
$$d = 17k - k^2$$

$\Rightarrow$ $k^2 + 8 = 17k - k^2$

$$2k^2 - 17k + 8 = 0$$
$$(2k - 1)(k - 8) = 0$$
$$k = \tfrac{1}{2} \text{ or } k = 8$$

10) A.P $a = k^2$ $d = k$ $k > 0$

5th term $= a + 4d = 41$

$k^2 + 4k = 41$

$k^2 + 4k - 41 = 0$

$k = -2 + 3\sqrt{5}$ or $\cancel{-2 - 3\sqrt{5}}$

$\underline{k = -2 + 3\sqrt{5}}$

Exercise 3B

Q11 AP $\underset{1st}{(k+1)} + \underset{2nd}{(2k+3)} + \underset{3rd}{(3k+5)} + \dots + 303$

$a = k+1$ n^{th} term $= a + (n-1)d$

$d = k+2$

n^{th} term $= k + 1 + (n-1)(k+2) = 303$

$(n-1)(k+2) = 303 - k - 1$

$n - 1 = \dfrac{302 - k}{k+2}$

$n = \dfrac{302 - k}{k+2} + 1$

b) $S_n = \frac{n}{2}(a + L)$

$= \dfrac{\left(\dfrac{302-k}{(k+2)} + 1\right)}{2}\left(k+1 + 303\right)$

$$= \left(\frac{302 - \cancel{k} + \cancel{k} + 2}{2(k+2)}\right)(k+304)$$

$$= \frac{152}{(k+2)}(k+304) = \frac{152k + 46208}{k+2}$$

---

c) $$\frac{152k + 46208}{k+2} = 2568$$

$$152k + 46208 = 2568k + 5136$$

$$41072 = 2416k$$

$$\frac{41072}{2416} = k$$

$$\underline{k = 17}$$