14. The first three terms of an arithmetic series are p, $5p - 8$, and $3p + 8$ respectively.

(a) Show that $p = 4$.

(2)

(b) Find the value of the 40th term of this series.

(3)

(c) Prove that the sum of the first n terms of the series is a perfect square.

(3)

(Total 8 marks)

$$d = 5p - 8 - p = 4p - 8$$

$$d = 3p + 8 - (5p - 8)$$
$$= 3p + 8 - 5p + 8$$
$$= -2p + 16$$

$$\therefore \quad 4p - 8 = -2p + 16$$

$$4p + 2p = 16 + 8$$

$$6p = 24$$

$$p = \frac{24}{6}$$

$$\underline{p = 4}$$

b) $\quad a = p = 4$

$$d = 4p - 8 = 4(4) - 8 = 8$$

$$40^{th} \text{ term} = a + 39d$$
$$= 4 + 39 \times 8$$
$$= 316$$

c) $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$$S_n = \frac{n}{2}\left(2(4) + (n-1)(8)\right)$$

$$= \frac{n}{2}\left(8 + 8n - 8\right)$$

$$= \frac{n}{2}\left(8n\right)$$

$$= 4n^2$$

$$= (2n)^2 \qquad \text{a perfect square}$$