Trig Eqns and Identities 2008-10

Question Number	Scheme	Marks	
6.	(a) $\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x$	M1	
	$=(2\cos^2 x-1)\cos x-(2\sin x\cos x)\sin x$	M1	
	$=(2\cos^2 x-1)\cos x-2(1-\cos^2 x)\cos x$ any correct expression	A1	
	$=4\cos^3 x-3\cos x$	A1	(4)
	(b)(i) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}=\dfrac{\cos^2 x+(1+\sin x)^2}{(1+\sin x)\cos x}$	M1	
	$=\dfrac{\cos^2 x+1+2\sin x+\sin^2 x}{(1+\sin x)\cos x}$	A1	
	$=\dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$	M1	
	$=\dfrac{2}{\cos x}=2\sec x$ * cso	A1	(4)
	(c) $\sec x=2$ *or* $\cos x=\dfrac{1}{2}$	M1	
	$x=\dfrac{\pi}{3},\dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1	(3)
			[11]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\text{cosec}^2\theta$ $\ast$ cso	A1 (2)
	Alternative for (a)	
	$1+\cot^2\theta=1+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$=\text{cosec}^2\theta$ $\ast$ cso	A1
	(b) $2\left(\text{cosec}^2\theta-1\right)-9\,\text{cosec}\,\theta=3$	M1
	$2\,\text{cosec}^2\theta-9\,\text{cosec}\,\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$(2\,\text{cosec}\,\theta+1)(\text{cosec}\,\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$	M1
	$\text{cosec}\,\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6) **[8]**

Question Number	Scheme		Marks	
6.	(a)(i) $\sin 3\theta = \sin(2\theta + \theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ ✱	cso	A1	**(4)**
	(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	**(5)**
	(b) $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\surd 3}{2} \times \frac{1}{\surd 2} - \frac{1}{2} \times \frac{1}{\surd 2}$		M1 A1	
	$= \frac{1}{4}\surd 6 - \frac{1}{4}\surd 2 = \frac{1}{4}(\surd 6 - \surd 2)$ ✱	cso	A1	**(4)**
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\surd 2} \times \frac{\surd 3}{2} - \frac{1}{\surd 2} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\surd 6 - \frac{1}{4}\surd 2 = \frac{1}{4}(\surd 6 - \surd 2)$ ✱	cso	A1	**(4)**
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\surd 3}{2}$			
	$\sin^2 15° = \frac{2 - \surd 3}{4}$		M1 A1	
	$\left(\frac{1}{4}(\surd 6 - \surd 2)\right)^2 = \frac{1}{16}(6 + 2 - 2\surd 12) = \frac{2 - \surd 3}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\surd 6 - \surd 2)$ ✱	cso	A1	**(4)**

Question Number	Scheme		Marks
Q2 (a)	$\cos^2\theta + \sin^2\theta = 1 \quad \left(\div \cos^2\theta\right)$		
	$\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso (2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \quad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$3\sec^2\theta + 4\sec\theta - 4 = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$		
	$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$		
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$	$\cos\theta = -\frac{1}{2}$	A1;
	$\alpha = 120^\circ$ or α = no solutions		
	$\theta_1 = 120^\circ$	120°	A1
	$\theta_2 = 240^\circ$	240° or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	(6)
			[8]

Question Number	Scheme		Marks
Q8	$\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$		
	Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\text{cosec}^2\theta = \pm 1 \pm \cot^2 \theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad$ or $\quad \cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad$ or $\quad \cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad$ or $\quad \cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

edexcel

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6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\not{2}\sin\theta\not{\cos\theta}}{\not{2}\cos\theta\not{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 **cso**
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	θ_1 = awrt 26.6°	A1
	θ_2 = awrt -153.4°	A1 $\sqrt{}$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1 $\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	