ALGEBRAIC AND PARTIAL FRACTIONS

1)

$$\frac{x^2 - 36}{x^2 - 11x + 30} \times \frac{25 - x^2}{Ax^2 + Bx + c} \times \frac{6x^2 + 7x - 3}{3x^2 + 17x - 6} \equiv \frac{x + 5}{6 - x}$$

$$\frac{\cancel{(x+6)}\cancel{(x-6)}}{\cancel{(x-5)}\cancel{(x-6)}} \times \frac{(5+x)\cancel{(5-x)}}{Ax^2 + Bx + c} \times \frac{\cancel{(3x-1)}(2x+3)}{\cancel{(3x-1)}\cancel{(x+6)}} \equiv \frac{x+5}{6-x}$$

$$\frac{(5+x)(2x+3)}{-1(Ax^2 + Bx + c)} \equiv \frac{x+5}{6-x}$$

$$\frac{2x+3}{-1(Ax^2 + Bx + c)} \equiv \frac{1}{6-x}$$

$$(2x+3)(6-x) = -1(Ax^2 + Bx + c)$$

$$12x + 18 - 2x^2 - 3x = -(Ax^2 + Bx + c)$$

$$-2x^2 + 9x + 18 = -(Ax^2 + Bx + c)$$

$$\Rightarrow \quad A = 2, \quad B = -9, \quad C = -18$$

2) $$\frac{6}{4x^2 + 8x - 5} \quad + \quad \frac{3x+1}{2x-1}$$

$$= \frac{6}{(2x-1)(2x+5)} \quad + \quad \frac{3x+1}{2x-1}$$

$$= \frac{6 + (3x+1)(2x+5)}{(2x-1)(2x+5)}$$

$$= \frac{6 + 6x^2 + 2x + 15x + 5}{(2x-1)(2x+5)} = \frac{6x^2 + 17x + 11}{(2x-1)(2x+5)}$$

$$= \frac{(6x+11)(x+1)}{(2x-1)(2x+5)}$$

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3) $\dfrac{6(x+7)}{(5x-1)(2x+5)} \equiv \dfrac{A}{5x-1} + \dfrac{B}{2x+5}$

$\qquad 6(x+7) \equiv A(2x+5) + B(5x-1)$

$x=\dfrac{1}{5} \qquad 6\left(\dfrac{36}{5}\right) = A\left(\dfrac{27}{5}\right)$

$\qquad\qquad\qquad 216 = 27A \qquad\qquad \Rightarrow A = 8$

$x=-\dfrac{5}{2}$

$\qquad\qquad 6\left(\dfrac{9}{2}\right) = B\left(-\dfrac{27}{2}\right)$

$\qquad\qquad\qquad 54 = -27B \qquad\qquad \Rightarrow B = -2$

$\qquad \dfrac{6(x+7)}{(5x-1)(2x+5)} \equiv \dfrac{8}{5x-1} - \dfrac{2}{2x+5}$

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4) $\quad f(x) = \dfrac{4x^2 + x - 23}{(x-3)(4-x)(x+5)} \equiv \dfrac{A}{x-3} + \dfrac{B}{4-x} + \dfrac{C}{x+5}$

$4x^2 + x - 23 \equiv A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x)$

$x=3$
$\qquad 4(3)^2 + 3 - 23 = A(4-3)(3+5)$

$\qquad\qquad 16 = 8A \qquad\qquad\qquad A = 2$

$x=4$
$\qquad 4(4)^2 + 4 - 23 = B(4-3)(4+5)$

$\qquad\qquad 45 = 9B \qquad\qquad\qquad B = 5$

$x=-5$
$\qquad 4(-5)^2 - 5 - 23 = C(-5-3)(4+5)$

$\qquad\qquad 72 = -72C \qquad\qquad\qquad C = -1$

5)

$$\frac{18x^2 - 98x + 78}{(x-4)^2(3x+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{3x+1}$$

$$18x^2 - 98x + 78 = A(x-4)(3x+1) + B(3x+1) + C(x-4)^2$$

$x = 4$

$$18(4)^2 - 98(4) + 78 = B(3(4)+1)$$

$$-26 = 13B \qquad\qquad \Rightarrow B = -2$$

$x = -\frac{1}{3}$

$$18\left(-\frac{1}{3}\right)^2 - 98\left(-\frac{1}{3}\right) + 78 = C\left(-\frac{1}{3} - 4\right)^2$$

$$\frac{338}{3} = \frac{169}{9}C \qquad\qquad \Rightarrow C = 6$$

$$\frac{338}{3} \times \frac{9}{169} = C$$

Coeff of x^2

$$18 = 3A + C$$

$$18 - 6 = 3A \qquad\qquad\qquad \Rightarrow A = 4$$

$$12 = 3A$$

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6)

$$\frac{x^3 + 8x^2 - 9x + 12}{x+6} = Ax^2 + Bx + C + \frac{D}{x+6}$$

$$
\begin{array}{r}
x^2 + 2x - 21 \\
x+6 \overline{\smash{\big)}\ x^3 + 8x^2 - 9x + 12} \\
\underline{x^3 + 6x^2} \\
2x^2 - 9x \\
\underline{2x^2 + 12x} \\
-21x + 12 \\
\underline{-21x - 126} \\
+138
\end{array}
$$

$$A = 1, \quad B = 2, \quad C = -21, \quad D = 138$$

ALGEBRAIC AND PARTIAL FRACTIONS   TEST 2

7) $f(x) = \dfrac{x^4 + 2x^3 - 29x^2 - 47x + 77}{x^2 - 2x - 15}$

$$
\begin{array}{r}
x^2 + 4x - 6 \\
x^2 - 2x - 15 \overline{\smash{\big)}\ x^4 + 2x^3 - 29x^2 - 47x + 77} \\
\underline{x^4 - 2x^3 - 15x^2} \\
4x^3 - 14x^2 - 47x \\
\underline{4x^3 - 8x^2 - 60x} \\
-6x^2 + 13x + 77 \\
\underline{-6x^2 + 12x + 90} \\
+x - 13
\end{array}
$$

$f(x) = x^2 + 4x - 6 + \dfrac{x - 13}{x^2 - 2x - 15}$

$\dfrac{x-13}{x^2 - 2x - 15} \equiv \dfrac{x-13}{(x-5)(x+3)} \equiv \dfrac{V}{x+3} + \dfrac{W}{x-5}$

$x - 13 \equiv V(x-5) + W(x+3)$

$x = 5$

$\qquad 5 - 13 = W(5+3)$

$\qquad -8 = 8W \qquad\qquad \Rightarrow W = -1$

$x = -3$

$\qquad -3 - 13 = V(-3-5)$

$\qquad -16 = -8V \qquad\qquad \Rightarrow V = 2$

$f(x) = x^2 + 4x - 6 + \dfrac{2}{x+3} - \dfrac{1}{x-5}$

$P = 1, \quad Q = 4, \quad R = -6, \quad V = 2, \quad W = -1$

ALGEBRAIC AND PARTIAL FRACTIONS

8) $5x^4 - 4x^3 + 17x^2 - 5x + 7 \equiv (Ax^2 + Bx + C)(x^2 + 2) + Dx + E$

$$5x^4 - 4x^3 + 17x^2 - 5x + 7 \equiv Ax^4 + Bx^3 + Cx^2$$
$$+ 2Ax^2 + 2Bx + 2C$$
$$+ Dx + E$$

coeff of x^4 $\qquad 5 = A$ $\qquad\qquad A = 5$

coeff of x^3 $\qquad -4 = B$ $\qquad\qquad B = -4$

coeff of x^2 $\qquad 17 = C + 2A$
$\qquad\qquad\qquad = C + 10$ $\qquad\qquad C = 7$

coeff of x $\qquad -5 = 2B + D$
$\qquad\qquad\qquad = -8 + D$ $\qquad\qquad D = 3$

coeff of constant $\qquad 7 = 2C + E$
$\qquad\qquad\qquad 7 = 14 + E$ $\qquad\qquad E = -7$

9) $f(x) = \dfrac{9x^2 + 25x + 16}{9x^2 - 16} \equiv A + \dfrac{B}{3x - 4} + \dfrac{C}{3x + 4}$

$9x^2 + 25x + 16 \equiv A(3x-4)(3x+4) + B(3x+4) + C(3x-4)$

$x = \frac{4}{3}$ $\quad 9\left(\frac{4}{3}\right)^2 + 25\left(\frac{4}{3}\right) + 16 = B\left(3\left(\frac{4}{3}\right) + 4\right)$

$\qquad\qquad \dfrac{196}{3} = 8B$ $\qquad\qquad B = \dfrac{49}{6}$

$x = -\frac{4}{3}$ $\quad 9\left(-\frac{4}{3}\right)^2 + 25\left(-\frac{4}{3}\right) + 16 = C\left(3\left(-\frac{4}{3}\right) - 4\right)$

$\qquad\qquad -\dfrac{4}{3} = -8C$ $\qquad\qquad C = \dfrac{1}{6}$

coeff of x^2 $\qquad 9 = 9A$ $\qquad\qquad A = 1$

$$f(x) = 1 + \dfrac{49}{6(3x-4)} + \dfrac{1}{6(3x+4)}$$