1. (a) $23,35.5$ (may be in the table)

B1 B1
2
(b) Width of 10 units is 4 cm so width of 5 units is $\underline{\mathbf{2 ~ c m}}$

B1

M1 A1 3
3

Height $=2.6 \times 4=\underline{\mathbf{1 0 . 4} \mathbf{~ c m}}$

## Note

M1 for their width $\times$ their height $=20.8$.
Without labels assume width first, height second and award marks accordingly.
(c) $\quad \sum \mathrm{f} x=1316.5 \Rightarrow \bar{x}=\frac{1316.5}{56}=\quad$ awrt $\underline{23.5}$

$$
\sum \mathrm{f} x^{2}=37378.25 \text { can be implied }
$$

So $\sigma=\sqrt{\frac{37378.25}{56}-\bar{x}^{2}}=$ awrt $\underline{\mathbf{1 0 . 7}} \quad$ allow $s=10.8 \quad$ M1 A1
5

## Note

$1^{\text {st }}$ M1 for reasonable attempt at $\sum x$ and /56
$2^{\text {nd }} \mathrm{M} 1$ for a method for $\sigma$ or $s, \sqrt{ }$ is required
Typical errors $\sum(\mathrm{f} x)^{2}=354806.3 \mathrm{M} 0, \sum \mathrm{f}^{2} x=13922.5 \mathrm{M} 0$ and $\left(\sum \mathrm{f} x\right)^{2}=1733172 \mathrm{M} 0$
Correct answers only, award full marks.
(d) $\quad Q_{2}=(20.5)+\frac{(28-21)}{11} \times 5=23.68 \ldots \quad$ awrt $\underline{23.7 \text { or } 23.9} \quad$ M1 A1 2

## Note

Use of $\sum \mathrm{f}(x-\bar{x})^{2}=$ awrt 6428.75 for B1
lcb can be $20,20.5$ or 21 , width can be 4 or 5 and the fraction part of the formula correct for M1 - Allow 28.5 in fraction that gives awrt 23.9 for M1A1
(e) $\quad Q_{3}-Q_{2}=5.6, \quad Q_{2}-Q_{1}=7.9 \quad$ (or $\left.\bar{x}<Q_{2}\right) \quad$ M1

$$
[7.9>5.6 \text { so }] \quad \text { negative skew } \quad \text { A1 } 2
$$

## Note

M1for attempting a test for skewness using quartiles or mean and median.
Provided median greater than 22.55 and less than 29.3 award for M 1 for $Q_{3}-Q_{2}<Q_{2}-Q_{1}$ without values as a valid reason.
SC Accept mean close to median and no skew oe for M1A1
2. (a) Median is 33
(b) $Q_{1}=24, Q_{3}=40, \mathrm{IQR}=16$

## Note

$1^{\text {st }} \mathrm{B} 1$ for $\mathrm{Q} 1=24$ and 2nd B1 for $Q_{3}=40$
$3^{\text {rd }}$ B1ft for their IQR based on their lower and upper quartile.

Calculation of range ( $40-7=33$ ) is B0B0B0
Answer only of IQR = 16 scores $3 / 3$. For any other answer we must see working in (b) or on stem and leaf diagram
(c) $Q_{1}-\mathrm{IQR}=24-16=8$

M1
A1ft

B1 1

B1 B1 B1ft
3

A typical error in (d) is to draw the lower whisker to 7, this can only score B1B0B0
3. (a) 2.75 or $2 \frac{3}{4}, 5.5$ or 5.50 or $5 \frac{1}{2}$

B1 B1 2
(b) Mean birth weight $=\frac{4841}{1500}=3.227 \dot{3} \quad$ awrt $3.23 \quad$ M1 A1 2

## Note

M1 for a correct expression for mean. Answer only scores both.
(c) Standard deviation $=\sqrt{\frac{15889.5}{1500}-\left(\frac{4841}{1500}\right)^{2}}=$
$0.421093 \ldots$ or $s=0.4212337 \ldots$
M1 A1ft A1
3

## Note

M1 for a correct expression (ft their mean) for sd or variance. Condone mis-labelling eg $s d=\ldots$
with no square root or no labelling
$1^{\text {st }}$ A1ft for a correct expression (ft their mean) including square root and no mis-labelling

Allow $1^{\text {st }} \mathrm{A} 1$ for $\sigma^{2}=0.177 \ldots \rightarrow \sigma=0.42 \ldots$
$2^{\text {nd }} A 1$ for awrt 0.421 . Answer only scores $3 / 3$
(d) $\quad Q_{2}=3.00+\frac{400}{820} \times 0.5=3.2457 \ldots . \quad$ (allow $403.5 \ldots$.

$$
\rightarrow 3.25)
$$

M1 A1
2

## Note

M1 for a correct expression (allow 403.5 i.e. use of $n+1$ ) but must have 3.00 , 820 and 0.5
A1 for awrt 3.25 provided M1 is scored.
NB 3.25 with no working scores $0 / 2$ as some candidates think mode is 3.25 .
(e) Mean(3.23)<Median(3.25)

Negative Skew

| (or very close) | B 1 ft |
| :--- | ---: |
| (or symmetrical) | dB 1 ft |

B1ft
dB1ft
2

## Note

$1^{\text {st }} \mathrm{B} 1 \mathrm{ft}$ for a comparison of their mean and median (may be in a formula but if $\pm$ (mean - median) is calculated that's OK. We are not checking
the value but the sign must be consistent.)
Also allow for use of quartiles provided correct
values seen: $Q_{1}=3.02, Q_{3}=3.47$
[They should get (0.22 $=$ ) $Q_{3}-Q_{2}<Q_{2}-Q_{1}(=0.23)$ and say (slight) negative skew or symmetric]
$2^{\text {nd }} \mathrm{dB} 1 \mathrm{ft}$ for a compatible comment based on their comparison.
Dependent upon a suitable, correct comparison.
Mention of "correlation" rather than "skewness"
loses this mark.
4. (a)


3 closed curves and 4 in centre M1
Evidence of subtraction M1
31,36,24 A1
41,17,11 A1
Labels on loops, 16 and box B1

## Note

$2^{\text {nd }}$ M1 There may be evidence of subtraction in "outer" portions, so with 4 in the centre then 35,4028 (instead of $31,36,24$ ) along with $33,9,3$ can score this mark but A0A0
N.B. This is a common error and their " 16 " becomes 28 but still scores B0 in part (a)
(b) $\mathrm{P}($ None of the 3 options $)=\frac{16}{180}=\frac{4}{45}$

## Note

B1ft for $\frac{16}{180}$ or any exact equivalent. Can ft their " 16 " from their box. If there is no value for their " 16 " in the box only allow this mark if they have shown some working.
(c) $\quad \mathrm{P}($ Networking only $)=\frac{17}{180}$

B1ft 1

## Note

B1ft ft their "17". Accept any exact equivalent
(d) $\mathrm{P}\left(\right.$ All 3 options/technician) $=\frac{4}{40}=\frac{1}{10}$

M1 A1 2

Note
If a probability greater than 1 is found in part
(d) score M0A0

M1 for clear sight of $\frac{\mathrm{P}(S \cap D \cap N)}{\mathrm{P}(S \cap N)}$ and an attempt at one of the probabilities, ft their values.
Allow P(all $3 \mid S \cap N)=\frac{4}{36}$ or $\frac{1}{9}$ to score M1 A0.
Allow a correct ft from their diagram to score M1A0 e.g. in $33,3,9$ case in (a): $\frac{4}{44}$ or $\frac{1}{11}$ is M1A0 A ratio of probabilities with a product of probabilities on top is M0, even with a correct formula.
A1 for $\frac{4}{40}$ or $\frac{1}{10}$ or an exact equivalent
Allow $\frac{4}{40}$ or $\frac{1}{10}$ to score both marks if this follows from their diagram, otherwise some explanation (method) is required.
5. (a) $1(\mathrm{~cm})$ B1 cao
(b) $10 \mathrm{~cm}^{2}$ represents 15
$10 / 15 \mathrm{~cm}^{2}$ represents $1 \quad$ or $1 \mathrm{~cm}^{2}$ represents 1.5
Therefore frequency of 9 is
$\frac{10}{15} \times 9$ or $\frac{9}{1.5} \quad$ Require $\times \frac{2}{3}$ or $\div 1.5$
height $=6$ (cm)

## Note

If 3(a) and 3(b) incorrect, but their
(a) $\times$ their (b) $=6$ then award B0M1A0

Alternative method:
$\mathrm{f} / \mathrm{cw}=15 / 6=2.5$ represented by 5 so factor x 2 award M1
So $f / c w=9 / 3=3$ represented by $3 \times 2=6$. Award A1.
6. (a) $Q_{2}=17+\left(\frac{60-58}{29}\right) \times 2$
$=17.1$ (17.2 if use 60.5)
awrt 17.1 (or17.2)
A1 2

## Note

Statement of $17+\frac{\text { freq into class }}{\text { class freq }} \times \mathrm{cw}$
and attempt to sub or $\frac{m-17}{19-17}=\frac{60(.5)-58}{87-58}$
or equivalent award M1
$\mathrm{cw}=2$ or 3 required for M1.
17.2 from cw $=3$ award A0.
(b) $\quad \sum f x=2055.5 \quad \sum f x^{2}=36500.25 \quad$ Exact answers can be seen below or implied by correct answers.

B1 B1
Evidence of attempt to use midpoints with
at least one correct
Mean $=17.129 \ldots \quad$ awrt $17.1 \quad$ B1
$\sigma=\sqrt{\frac{36500.25}{120}-\left(\frac{2055.5}{120}\right)^{2}}$
$=3.28$ ( $s=3.294$ )
awrt 3.3
A1 6

## Note

Correct $\sum \mathrm{fx}$ and $\sum \mathrm{fx} x^{2}$ can be seen in working
for both B1s
Midpoints seen in table and used in calculation award M1
Require complete correct formula including use of square root and attempt to sub for M1. No formula stated then numbers as above or follow from (b) for M1
$\left(\sum f x\right)^{2}, \sum(f x)^{2}$ or $\sum f^{2} x$ used instead of $\sum f x^{2}$
in sd award M0
Correct answers only with no working award $2 / 2$ and 6/6
(c) $\frac{3(17.129-17.1379 \ldots)}{3.28}=-0.00802 \quad$ Accept 0 or awrt $0.0 \quad$ M1 A1

No skew/ slight skew

## Note

Sub in their values into given formula for M1
(d) The skewness is very small. Possible.

B1 B1dep
2

## Note

No skew / slight skew / 'Distribution is almost symmetrical' / 'Mean approximately equal to median' or equivalent award first B1. Don't award second B1 if this is not the case. Second statement should imply 'Greg's suggestion that a normal distribution is suitable is possible’ for second B1 dep.

If B0 awarded for comment in (c).and (d) incorrect, allow follow through from the comment in (c).
7. (a) $\mathrm{Q}_{2}=53, \quad Q_{1}=35, \quad Q_{3}=60$ B1, B1, B1

3

## Note

$1^{\text {st }}$ B1 for median
$2^{\text {nd }} \mathrm{B} 1$ for lower quartile
$3^{\text {rd }} \mathrm{B} 1$ for upper quartile
(b) $Q_{3}-Q_{1}=25 \Rightarrow Q_{1}-1.5 \times 25=-2.5$ (no outlier)

$$
Q_{3}+1.5 \times 25=97.5(\text { so } 110 \text { is an outlier })
$$

## Note

M1 for attempt to find one limit
A1 for both limits found and correct. No explicit comment about outliers needed.
(c)


M1
A1ft
A1ft
3

## Note

M1 for a box and two whiskers
$1^{\text {st }}$ A1ft for correct position of box, median and quartiles. Follow through their values.
$2^{\text {nd }} \mathrm{A} 1 \mathrm{ft}$ for 17 and 77 or "their" 97.5 and *. If 110 is not an outlier then score A0 here. Penalise no gap between end of whisker and outlier. Must label outlier, needn't be with * .
Accuracy should be within the correct square so 97 or 98 will do for 97.5
(d) $\quad \sum y=461, \sum y^{2}=24219 \therefore \mathrm{~S}_{y y}=24219-\frac{461^{2}}{10},=2966.9\left(^{*}\right) \quad$ B1, B1,

## Note

$1^{\text {st }} \mathrm{B} 1$ for $\sum y$ N.B. $\left(\sum y\right)^{2}=212521$ and can imply this mark
$2^{\text {nd }}$ B1 for $\sum y^{2}$ or at least three correct terms of $\sum(y-\bar{y})^{2}$ seen.
$3^{\text {rd }} \mathrm{B} 1$ for complete correct expression seen leading to 2966.9. So all 10
terms of $\sum(y-\bar{y})^{2}$
(e) $\quad r=\frac{-18.3}{\sqrt{3463.6 \times 2966.9}}$ or $\frac{-18.3}{3205.64 \ldots}=-0.0057$

AWRT -0.006 or $-6 \times 10^{-3}$

## Note

M1 for attempt at correct expression for $r$. Can ft their $\mathrm{S}_{y y}$ for M 1 .
(f) $\quad r$ suggests correlation is close to zero so parent's claim is not justified $\quad$ B1 $\quad 1$

## Note

B1 for comment rejecting parent's claim on basis of weak or zero correlation
Typical error is "negative correlation so comment is true" which scores B0
Weak negative or weak positive correlation is OK as the basis for their rejection.
8. (a) $8-10$ hours: width $=10.5-7.5=3$ represented by 1.5 cm $16-25$ hours: width $=25.5-15.5=10$ so represented by $\underline{5 \mathrm{~cm}}$
$8-10$ hours: height $=\mathrm{fd}=18 / 3=6$ represented by 3 cm
$16-25$ hours: height $=\mathrm{fd}=15 / 10=1.5$ represented by $\underline{0.75 \mathrm{~cm}}$

## Note

M1 For attempting both frequency densities

$$
\frac{18}{3}(=6) \text { and } \frac{15}{10} \text {, and } \frac{15}{10} \times \text { SF, where } S F \neq 1 \text { NB Wrong class }
$$

widths (2 and 9) gives $\frac{h}{1.66 \ldots . .}=\frac{3}{9} \rightarrow h=\frac{5}{9}$ or $0.55 \ldots$
and scores M1A0
(b) $\quad Q_{2}=7.5+\frac{(52-36)}{18} \times 3=10.2$
$Q_{1}=5.5+\frac{(26-20)}{16} \times 2[=6.25$ or 6.3$]$ or $5.5+\frac{(26.25-20)}{16} \times 2[=6.3]$
$Q_{3}=10.5+\frac{(78-54)}{25} \times 5[=15.3]$ or $10.5+\frac{(78.75-54)}{25} \times 5[=15.45 \backslash 15.5] \quad \mathrm{A} 1$
$\operatorname{IQR}=(15.3-6.3)=\underline{9}$
A1ft

## Note

M1 for identifying correct interval and a correct fraction e.g. $\frac{\frac{1}{2}(104)-36}{18}$. Condone 52.5 or 53
$1^{\text {st }} \mathrm{A} 1 \quad$ for 10.2 for median. Using $(n+1)$ allow awrt 10.3

## NB:

$2^{\text {nd }}$ A1 for a correct expression for either $Q_{1}$ or $Q_{3}$ (allow 26.25 and 78.75)

Must see
$3^{\text {rd }} \mathrm{A} 1 \quad$ for correct expressions for both $Q_{1}$ and $Q_{3} \underline{\text { some }}$
$4^{\text {th }} \mathrm{A} 1 \mathrm{ft}$ for IQR, ft their quartiles. Using $(\mathrm{n}+1)$ gives 6.28 and 15.45
method
$\begin{array}{llll}\text { (c) } \sum f x=1333.5 \Rightarrow & \bar{x}=\frac{1333.5}{104}= & \text { AWRT } \underline{12.8} & \text { M1 A1 } \\ & \\ \sum f x^{2}=27254 \Rightarrow \sigma_{x}=\sqrt{\frac{27254}{104}-\bar{x}^{2}}=\sqrt{262.05-\bar{x}^{2}} & \text { AWRT 9.88 } & \text { M1 A1 } & 4\end{array}$

## Note

1st M1 for attempting $\sum f x$ and $\bar{x}$
2nd M1 for attempting $\sum f x^{2}$ and $\sigma_{x}, \sqrt{ }$ is needed for M1.
Allow s = awrt 9.93
(d) $Q_{3}-Q_{2}[=5.1]>Q_{2}-Q_{1}[=3.9]$ or $Q_{2}<\bar{x} \quad$ B1ft dB1 2

## Note

$1^{\text {st }}$ B1ft for suitable test, values need not be seen but statement must be compatible with values used. Follow through their values
$2^{\text {nd }} \mathrm{dB} 1$ Dependent upon their test showing positive and for stating positive skew If their test shows negative skew they can score $1^{\text {st }} \mathrm{B} 1$ but lose the second
(e) So data is positively skew

Use median and IQR, B1 since data is skewed or not affected by extreme values or outliers B1 B1 2

## Note

$1^{\text {st }}$ B1 for choosing median and IQR. Must mention both. \} Award independently
$2^{\text {nd }} \mathrm{B} 1$ for suitable reason \}
e.g. "use median because data is skewed" scores B0B1 since IQR is not mentioned
9. (a)


Bracketed probabilities not required.
(b) $\mathrm{P}($ Positive Test $)=0.02 \times 0.95+0.98 \times 0.03$
$=0.0484$
M1A1ft

M1 for sum of two products, at least one correct from their diagram
A1ft follows from the probabilities on their tree
A1 for correct answer only or $\frac{121}{2500}$
(c) $\quad \mathrm{P}($ Do not have disease $\mid$ Postive test $)=\frac{0.98 \times 0.03}{0.0484}$
$=0.607438 \ldots$ awrt 0.607
A1 2
M1 for condirtional probability with numerator following from their tree and denominator their answer to part (b).

A1 also for $\frac{147}{242}$.
(d) Test not very useful OR

High probability of not having the disease for a person with a positive test

B1 1
[9]
10. (a) 50

B1 1
(b) $\quad Q_{1}=45$
$Q_{2}=50.5$
$Q_{3}=63$
(c) Mean $=\frac{1469}{28}=52.464286 .$.
$\mathbf{S d}=\sqrt{\frac{81213}{28}-\left(\frac{1469}{28}\right)^{2}}$
$=12.164 \ldots$ or $12.387216 \ldots$ for divisor $n-1$ awrt 12.2 or 12.4

M1for their 1469 between 1300 and 1600, divided by 28,
A1 for awrt 52.5 ..
Please note this is B1B1 on Epen
M1 use of correct formula including sq root
A1 awrt 12.2 or 12.4
Correct answers with no working award full marks.
(d) $\frac{52.46 . .-50}{s d}=$ awrt 0.20 or 0.21

M1 for their values correctly substituted
A1 Accept 0.2 as a special case of awrt 0.20 with 0 missing
(e) 1. mode/median/mean Balmoral>mode/median/mean Abbey
2. Balmoral sd < Abbey sd or similar sd or correct comment from their values,
Balmoral range<Abbey range, Balmoral IQR>Abbey IQR or similar IQR
3. Balmoral positive skew or almost symmetrical AND Abbey negative skew, Balmoral is less skew than Abbey or correct comment from their value in (d)
4. Balmoral residents generally older than Abbey residents or equivalent.
Only one comment of each type max 3 marks

ONLY

Technical terms required in correct context in lines 1 to 3
e.g. 'average’ and 'spread’ B0

1 correct comment B1B0B0
2 correct comments B1B1B0
3 correct comments B1B1B1
11. (a)

$\begin{array}{lcc}3 \text { closed intersecting curves with labels } & \text { M1 } \\ 100100,30 & \text { A1 } & \\ 12,10,3,25 & \text { A1 } & \\ \text { Box } & \text { B1 } & 4\end{array}$
20 not required.
Fractions and exact equivalent decimals or percentages.
(b) $\mathrm{P}($ Substance $C)=\frac{100+100+10+25}{300}=\frac{235}{300}=\frac{47}{60}$ or exact equivalent M1A1ft 2

M1 For adding their positive values in $C$ and finding a probability
A1ft for correct answer or answer from their working
(c) $\mathrm{P}($ All $3 \mid A)=\frac{10}{30+3+10+100}=\frac{10}{143}$ or exact equivalent $\quad$ M1A1ft 2

M1 their 10 divided by their sum of values in $A$
A1ft for correct answer or answer from their working
(d) $\mathrm{P}($ Universal donor $)=\frac{20}{300}=\frac{1}{15}$ or exact equivalent

M1 for 'their 20' divided by 300
A1 correct answer only
12. (a) mean is $\frac{2757}{12},=229.75 \quad$ AWRT 230 M1, A1
sd is $\sqrt{\frac{724961}{12}-(229.75)^{2}},=87.34045 \quad$ AWRT 87.3 M1, A1 4
[Accept $s=$ AWRT 91.2]
$1^{\text {st }}$ M1 for using $\frac{\sum x}{n}$ with a credible numerator and $n=12$.
$2^{\text {nd }} \mathrm{M} 1$ for using a correct formula, root required but can ft their mean
Use of $s=\sqrt{8321.84 \ldots}=91.22 \ldots$ is OK for M1 A1 here.
Answers only from a calculator in (a) can score full marks
(b) Ordered list is:

125, 160, 169, 171, 175, 186, 210, 243, 250, 258, 390, 420
$Q_{2}=\frac{1}{2}(186+210)=198$ B1
$Q_{1}=\frac{1}{2}(169+171)=170$ B1
$Q_{3}=\frac{1}{2}(250+258)=254$
$1^{\text {st }} \mathrm{B} 1$ for median $=198$ only, $2^{\text {nd }} \mathrm{B} 1$ for lower quartile $3^{\text {rd }} \mathrm{B} 1$ for upper quartile
S.C. If all $\mathrm{Q}_{1}$ and $\mathrm{Q}_{3}$ are incorrect but an ordered list (with $\geq 6$ correctly placed) is seen and used then award B0B1 as a special case for these last two marks.
(c) $Q_{3}+1.5\left(Q_{3}-Q_{1}\right)=254+1.5(254-170),=380$

Accept AWRT (370-392) M1, A1
Patients $F$ (420) and $B$ (390) are outliers.
B1ftB1ft
M1 for a clear attempt using their quartiles in given formula,
A1 for any value in the range $370-392$
$1^{\text {st }} \mathrm{B} 1 \mathrm{ft}$ for any one correct decision about $B$ or $F-\mathrm{ft}$ their limit in range $(258,420)$
$2^{\text {nd }}$ B1ft for correct decision about both $F$ and $B-\mathrm{ft}$ their limit in range $(258,420)$
If more points are given score B0 here for the second B mark. (Can score M0A0B1B1 here)
(d) $\frac{Q_{1}-2 Q_{2}+Q_{3}}{Q_{3}-Q_{1}}=\frac{170-2 \times 198+254}{254-170},=0 . \dot{3}$

AWRT 0.33 M1, A1
Positive skew.
A1ft
3
M1 for an attempt to use their figures in the correct formula - must be seen
( $\geq 2$ correct substitutions)
$1^{\text {st }}$ A1 for AWRT 0.33
$2^{\text {nd }} \mathrm{A} 1 \mathrm{ft}$ for positive skew. Follow through their value/sign of skewness.
Ignore any further calculations.
"positive correlation" scores A0
13.

| Width | 1 | 1 | 4 | 2 | 3 | 5 | 3 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Freq. Density | 6 | 7 | 2 | 6 | 5.5 | 2 | 1.5 | 0.5 |

$0.5 \times 12$ or 6
Total area is $(1 \times 6)+(1 \times 7)+(4 \times 2)+\ldots,=70$
$(90.5-78.5) \times \frac{1}{2} \times \frac{140}{\text { their } 70}$

[^0]$1^{\text {st }}$ M1 for attempt at width of the correct bar (90.5-78.5)
[Maybe on histogram or in table]
$1^{\text {st }} \mathrm{A} 1$ for $0.5 \times 12$ or 6 (may be seen on the histogram). Must be related to the area of the bar above 78.5-90.5.
$2^{\text {nd }}$ M1 for attempting area of correct bar $\times \frac{140}{\text { their } 70}$
B1 for 70 seen anywhere in their working
$2^{\text {nd }}$ A1 for correct answer of 12 .

Minimum working required is $2 \times 0.5 \times 12$ where the 2 should come
from $\frac{140}{70}$
Beware 90.5-78.5 = 12 (this scores M1A0M0B0A0)
Common answer is $0.5 \times 12=6$ (this scores M1A1M0B0A0)
If unsure send to review e.g. $2 \times 0.5 \times 12=12$ without 70 being seen
14. (a) $\frac{1}{2}$

B1 1

Accept $50 \%$ or half or $0.5>$
Units not required.
(b) 54

Correct answers only.
Units not required.
(c) + is an 'oulier' or 'extreme value'

B1
Any heavy musical instrument or a statement that the instrument is heavy

B1 1
'Anomaly' only award B0
Accept ' 85 kg was heaviest instrument on the trip' or equivalent for second B1.
Examples of common acceptable instruments; double bass, cello, harp, piano, drums, tuba
Examples of common unacceptable instruments: violin, viola, trombone, trumpet, French horn, guitar
(d) $Q_{3}-Q_{2}=Q_{2}-Q$
so symmetrical or no skew Dependent - only award if B1 above
'Quartiles equidistant from median' or equivalent award B1 then symmetrical or no skew for B1
Alternative:
'Positive tail is longer than negative tail' or 'median closer to lowest value' or equivalent so slight positive skew.
B0 for 'evenly' etc. instead or 'symmetrical'
B0 for 'normal' only
(e) $\mathrm{P}(W<54)=0.75($ or $\mathrm{p}(W>54)=0.25)$ or correctly labelled and
shaded diagram

$$
\begin{aligned}
& \frac{54-45}{\sigma}=0.67 \\
& \sigma=13.43 \ldots .
\end{aligned}
$$

## M1B1

A1 4

## Please note that $B$ mark appears first on ePEN

First line might be missing so first M1 can be implied by second.
Second M1 for standardising with sigma and equating to z value
NB Using 0.7734 should not be awrded second M1
Anything which rounds to 0.67 for B1.
Accept 0.675 if to 3sf obtained by interpolation
Anything that rounds to 13.3. - 13.4 for A1.
15. (a) Use overlay

B2
2
Points B2, within 1 small square of correct point, subtract 1 mark each error minimum 0 .
(b) $S_{x y}=28750-\frac{315 \times 620}{8}=4337.5 * *$ answer given** so award for method
$S_{x x}=15225-\frac{315^{2}}{8}=2821.875$
M1A1
3

Anything that rounds to 2820 for A1
(c) $\quad b=\frac{4377.5}{S_{x x}},=1.537 \ldots=1.5$

M1, A1
$a=\bar{y}-b \bar{x}=\frac{620}{8}-b \frac{315}{8}=16.97 \ldots=17.0$
M1, A1
4

Anything that rounds to 1.5 and 17.0 (accept 17)
(d) Use overlay $\quad \mathrm{B} 1 \mathrm{ft}$ B1ft

Follow through for the intercept for first B1.
Correct slope of straight line for second B1.
(e) Brand D. B1
since a long way above / from the line (dependent upon 'Brand D' above) B1 Using line: $y=17+35 \times 1.5=69.5$

M1A1
Anything that rounds to 69 p -71 p for final A1.
Reading from graph is acceptable for M1A1.
If value read from graph at $x=35$ is answer given but out of range, then award M1A0.
16. (a) $18-25$ group, area $=7 \times 5=35$

B1
$25-40$ group, area $=15 \times 1=15$
B1
(b) $(25-20) \times 5+(40-25) \times 1=40$ M1A1
$5 \times 5$ is enough evidence of method for M1.
Condone 19.5, 20.5 instead of 20 etc.
Award 2 if 40 seen.
(c) Mid points are $7.5,12,16,21.5,32.5$ M1
$\Sigma \mathrm{f}=100$ B1
$\frac{\sum \mathrm{f} t}{\sum \mathrm{f}}=\frac{1891}{100}=18.91$
M1A1
4

Look for working for this question in part (d) too.
Use of some mid-points, at least 3 correct for M1. These may be tabulated in (d).
Their $\frac{\sum \mathrm{f} t}{\sum \mathrm{f}}$ for M1 and anything that rounds to 18.9 for A1.
(d) $\quad \sigma_{t}=\sqrt{\frac{41033}{100}-\bar{t}^{2}}$
$\sqrt{\frac{n}{n-1}\left(\frac{41033}{100}-t^{2}\right)}$ alternative OK
$\sigma_{t}=\sqrt{52.74 \ldots}=7.26$
A1 3
Clear attempt at $\frac{41033}{100}-\bar{t}^{2}$ or $\frac{n}{n-1}\left(\frac{41033}{100}-\bar{t}^{2}\right)$ alternative
for first M1.
They may use their $\bar{t}$ and gain the method mark.
Square root of above for second M1
Anything that rounds to 7.3 for A1.
(e) $\quad Q_{2}=18$ or 18.1 if $(n+1)$ used
$Q_{1}=10+\frac{15}{16} \times 4=13.75$ or 15.25 numerator gives 13.8125

Clear attempt at either quartile for M1
These will take the form 'their lower limit' + correct fraction $\times$ 'their class width'.
Anything that rounds to 13.8 for lower quartile.
23 or anything that rounds to 23.2 dependent upon method used.
(f) $0.376 \ldots$ B1

Positive skew B1ft
Anything that rounds to 0.38 for B1 or 0.33 for B1 if $(n+1)$ used.
Correct answer or correct statement that follows from their value for B1.
17. (a) Positive skew
(both bits)
B1 1
(b) $19.5+\frac{(60-29)}{43} \times 10,=26.7093 \ldots$.
awrt 26.7 M1, A1 2
(N.B. Use of 60.5 gives $26.825 \ldots$ so allow awrt 26.8)

M1 for $(19.5$ or 20$)+\frac{(60-29)}{43} \times 10$ or better.
Allow 60.5 giving awrt 26.8 for M1A1
Allow their $0.5 n$ [or $0.5(n+1)]$ instead of 60 [or 60.5 ] for M1.
(c) $\mu=\frac{3550}{120}=29.5833 \ldots$ or $29 \frac{7}{12}$
awrt $\underline{29.6}$
B1
$\sigma^{2}=\frac{138020}{120}-\mu^{2}$ or $\sigma=\sqrt{\frac{138020}{120}-\mu^{2}}$
A1 3
M1 for a correct expression for $\sigma, \sigma^{2}, s$ or $s^{2}$.
NB $\sigma^{2}=274.99$ and $s^{2}=277.30$
Condone poor notation if answer is awrt16.6 (or 16.7 for $s$ )
(d) $\frac{3(29.6-26.7)}{16.6}$
$=0.52 \ldots \quad$ awrt $\mathbf{0 . 5 2 0}$ (or with $s$ awrt 0.518 )
M1A1ft
A1 3
(N.B. 60.5 in (b) ...awrt 0.499 [or with $s$ awrt 0.497])

M1 for attempt to use this formula using their values to any accuracy. Condone missing 3.
$1^{\text {st }} \mathrm{A} 1 \mathrm{ft}$ for using their values to at least 3sf Must have the 3 .
$2^{\text {nd }}$ A1 for using accurate enough values to get awrt 0.520
(or 0.518 if using $s$ )
NB Using only 3 sf gives 0.524 and scores M1A1A0

(f) Use Median
$\begin{aligned} & \text { Since the data is skewed or less affected by outliers/extreme values }\end{aligned}$
$\begin{aligned} & 2^{\text {nd }} \text { B1 is dependent upon choosing median. }\end{aligned}$
dB1 $\begin{aligned} & \text { B1 } \\ & \text { (g) If the data are symmetrical or skewness is zero or } \\ & \begin{array}{ll}\text { normal/uniform distribution } \\ \text { ("mean =median" or "no outliers" or "evenly distributed" all score B0) }\end{array}\end{aligned}$
[14]
18. (a) Time is a continuous variable or data is in a grouped frequency table B1 1
(b) Area is proportional to frequency or $A \propto f$ or $A=k f$

B1 1
$1^{\text {st }} \mathrm{B} 1$ for one of these correct statements.
"Area proportional to frequency density" or
"Area = frequency" is B0
(c) $3.6 \times 2=0.8 \times 9$

M1
1 child represented by 0.8
$1^{\text {st }}$ M1 for a correct combination of any 2 of the 4 numbers:
3.6, 2, 0.8 and 9
e.g. $3.6 \times 2$ or $\frac{3.6}{0.8}$ or $\frac{0.8}{2}$ etc BUT e.g. $\frac{3.6}{2}$ is M0
$2^{\text {nd }}$ M1 dependent on $1^{\text {st }}$ M1 and for a correct combination of 3 numbers leading to $4^{\text {th }}$.
May be in separate stages but must see all 4 numbers
A1cso for fully correct solution.
Both Ms scored, no false working seen and comment required.
(d) $($ Total $)=\frac{24}{0.8},=\underline{\mathbf{3 0}}$

M1, A1 2

M1 for $\frac{24}{0.8}$ seen or implied.
19. (a) Indicates max / median / min / upper quartile / lower quartile (2 or more) B1 Indicates outliers (or equivalent description) B1
Illustrates skewness (or equivalent description e.g. shape) B1
Allows comparisons
Indicates range / IQR / spread
Any 3 rows
(b) (i) 37 (minutes) B1
(ii) Upper quartile or $\mathrm{Q}_{3}$ or third quartile or $75^{\text {th }}$ percentage or $\mathrm{P}_{75} \quad \mathrm{~B} 12$
(c) outliers

How to calculate correctly
'Observation that are very different from the other observations and need to be treated with caution’

These two children probably walked / took a lot longer
B1 2

$$
\text { Any } 2
$$

(d)


Box \& median \& whiskers M1
Sensible scale B1
30, 37, 50 B1
25, 55 B1
(e) Children from school A generally took less time
$50 \%$ of $\mathrm{B} \leq 37$ mins, $75 \%$ of $\mathrm{A}<37$ mins (similarly for 30 )
B1

Median / Q1 / Q3 / of A < median / Q1 / Q3 / of (1 or more)
B1
A has outliers, (B does not)
B1 4
Both positive skew
IQR of $\mathrm{A}<\mathrm{IQR}$ of B , range of $\mathrm{A}>$ range of B Any correct 4 lines
20. (a) $P$ (both longer than 24.5$)=\frac{11}{55} \times \frac{10}{54}=\frac{1}{27}$ or $0: 03 \dot{7}$ or 0.037

2 fracs $\times \mathrm{w} /$ o rep
M1 A1 2
awrt 0.037
(b) Estimate of mean time spent on their conversation is

$$
\begin{array}{rlr}
\bar{x}=\frac{1060}{55}= & 19 \frac{3}{11} \text { or } 19 . \dot{2} \dot{7} \text { or } 19.3 & \text { M1 A1 } 2 \\
& 1060 / \text { total, awrt } 19.3 \text { or } 19 \text { mins } 16 s &
\end{array}
$$

(c) $\frac{1060+\sum f y}{80}=21$
$21 \times 80=1680$
$\begin{array}{rlr}\sum f y=620 & \text { M1 } \\ \text { Subtracting 'their } 1060 \text { ' } & \end{array}$
$\therefore \bar{y}=\frac{620}{25}=24.8$
M1 A1 4
Dividing their 620 by 25

| (d) $\begin{array}{l}\text { Increase in mean value } \\ \text { Length of conversation increased considerably } \\ \text { During } 25 \text { weeks relative to } 55 \text { weeks } \\ \quad \text { Context- } \text { ft only from comment above }\end{array}$ | B1 |
| :--- | :---: |

21. (a) Mode is 56

B1 1
(b) $\mathrm{Q}_{1}=35, \mathrm{Q}_{2}=52, \mathrm{Q}_{3}=60$

B1, B1, B1 3
(c) $\bar{x}=\frac{1335}{27}=49.4$ or $49 \frac{4}{9}$
exact or awrt 49.4
$\sigma^{2}=\frac{71801}{27}-\left(\frac{1335}{27}\right)^{2}=214.5432 \ldots$
$\sigma=14.6$ or 14.9
awrt $14.6(5)$ or 14.9
A1 4
(d) $\frac{49.4-56}{14.6}=-0.448 \quad$ awrt range -0.44 to $-0.46 \quad$ M1A1 2
(e) For negative skew;

Mean<median<mode (49.4<52<56 not required) $\mathrm{Q}_{3}-\mathrm{Q}_{2}<\mathrm{Q}_{2}-\mathrm{Q}_{1}$
M1

2 compared correctly A1 3 compared correctly M1

8 and 17
Accept other valid reason eg. 3(mean-median)/sd as alt for M1 A1
8 and 17
22. (a) Distance is a continuous.

B1 1
continuous
(b) F.D $=$ freq/class width $\Rightarrow 0.8,3.8,5.3,3.7,0.75,0.1$
or the same multiple of
M1 A1 2
(c) $\mathrm{Q}_{2}=50.5+\frac{(67-23)}{53} \times 10=58.8$
awrt 58.8/58.9
$\begin{array}{rlrl}\mathrm{Q}_{1}=52.48 ; \mathrm{Q}_{3}=67.12 & \text { A1 A1 } & 4 \\ & \text { Special case: no working B1 B1 B1 (三A's on the epen) } & & \end{array}$
(d) $\bar{x}=\frac{8379.5}{134}=62.5335 \ldots$

B1
awrt 62.5
$s=\sqrt{\frac{557489.75}{134}-\left(\frac{8379.5}{134}\right)^{2}} \quad \quad$ M1 A1ft
$s=15.8089 \ldots\left(S_{n-1}=15.86825 \ldots\right) \quad$ A1 4
awrt 15.8 (15.9)
Special case: answer only B1 B1 (三A's on the epen)
(e) $\frac{Q_{3}-2 Q_{2}+Q_{1}}{Q_{3}-Q_{1}}=\frac{67.12-2 \times 58.8+52.48}{67.12-52.48}$
subst their $Q_{1}, Q_{2} \& Q_{3}$ need to show working for Alft and have reasonable values for quartiles

$$
\begin{array}{rlrl}
= & 0.1366 \Rightarrow & ;+ \text { ve skew } & \text { A1; B1 } 4 \\
& \text { awrt } 0.14 &
\end{array}
$$

B1 1
23. (a) $1.5\left(\mathrm{Q}_{3}-\mathrm{Q}_{1}\right)=1.5(28-12)=24$ B1
may be implied
$\mathrm{Q}_{3}+24=52 \Rightarrow 63$ is outlier
Att Q3 + ... or Q1 - ..., M1
52 and -12 or 0 or evidence of no lower outliers A1
$\mathrm{Q}_{1}-24<0 \Rightarrow$ no outliers
63 is an outlier
M1
A1
A1 7
(b) Distribution is +ve skew; $\mathrm{Q}_{2}-\mathrm{Q}_{1}(5)<\mathrm{Q}_{3}-\mathrm{Q}_{2}(11)$

B1; B1 2
(c) Many delays are small so passengers should find these acceptable or sensible comment in the context of the question.
24. (a) $\mathrm{Q}_{1}=33, \mathrm{Q}_{2}=41, \mathrm{Q}_{3}=52$
(b)

(c) Median of Northcliffe is greater than median of Seaview.

Upper quartiles are the same
IQR of Northcliffe is less than IQR of Seaview
Northcliffe positive skew, Seaview negative skew
Northcliffe symmetrical, Seaview positive skew (quartiles)
Range of Seaview greater than range of Northcliffe
any 3 acceptable comments
$\begin{array}{ll}\text { (d) On } 75 \% \text { of the nights that month } & \text { B1 } \\ \text { both had no more than } 52 \text { caravans on site. } & \text { B1 } 2\end{array}$
25. (a) $a=202, b=202, c=233$

B1,B1,B1
3
(b) $\quad Q_{1}-1.5\left(Q_{3}-Q_{1}\right)=191-1.5(221-191)=146$,
$Q_{3}=1.5\left(Q_{3}-Q_{1}\right)=221+1.5(221-191)=266$
attempt at one calculation, 146, 266
M1A1A1
$\Rightarrow 269$ is an outlier
269 A1dep


Scale and 'miles' B1
Box with two whiskers M1
191, their median, 221
A1ft
180,266 or 263,269
(c) Keith: $Q_{2}-Q_{1}=11, Q_{3}-Q_{2}=19 \Rightarrow$ positive skew one calc, +ve skewM1,A1

Asif: $Q_{2}-Q_{1}=16, Q_{3}-Q_{2}=15 \Rightarrow$ almost symm or slight -ve skew A1 3
26. (a) Time data is a continuous variable

B1 1
(b) 39.5, 44.5 both

B1 1
(c)


[^1]27. (a)
\[

$$
\begin{array}{lc}
\text { (i) } \begin{array}{lc}
\bar{x}=\frac{270}{16}=\underline{16.875} & \text { B1 } \\
16.875,167 / 716.9 ; 16.88 & \\
\text { s.d. }=\sqrt{\frac{4578}{16}-16.875^{2}} & \mathrm{M} 1 \\
\frac{\sum x^{2}}{16}-\bar{x}^{2} \& \sqrt{ } \\
\text { All correct } & \mathrm{A} 1 \mathrm{ft} \\
=\underline{\text { AWRT 1.17 }} \begin{array}{l}
\text { SR: No working B1 only }
\end{array} & \mathrm{Al}
\end{array}
\end{array}
$$
\]

(ii) Mean $\%$ attendance $=\frac{16.875}{18} \times 100(=93.75)$

B1 ft 5
cao
(b)

|  | First 4\|1 means 14 |  | Second $1 \mid 8$ means 18 |  |
| :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 4 | 1 | 444 | $(3)$ |
| $(1)$ | 5 | 1 | 5555 | $(4)$ |
| $(3)$ | 666 | 1 | 666 | $(3)$ |
| $(5)$ | 77777 | 1 | 7 | $(1)$ |
| $(6)$ | 8888888 | 1 | 888 | $(3)$ |
| $(0)$ |  | 1 | 9 | $(1)$ |
| $(0)$ |  | 2 | 0 | $(1)$ |

Both Labels and 1 key
Back-to-back S and L (ignore totals)
Sensible splits of 1
First-correct
Second - correct
(c)

|  | Mode | Median | IQR |
| :--- | :---: | :---: | :---: |
| First (F) | 18 | 17 | 2 |
| Second (S) | 15 | 16 | 3 |

(d) Median $_{\mathrm{S}}<$ Median $_{\mathrm{F}}$; Mode $_{\mathrm{F}}>$ Mode $_{\mathrm{S}}$; $\mathrm{IQR}_{\mathrm{s}}>\mathrm{IQR}_{\mathrm{F}}$; Only 1 student attends all classes in second; Mean\% ${ }_{\mathrm{F}}>$ Mean\%s

Any THREE sensible comments
28. (a)

| Sales | No. of days | Class width | Frequency density |
| :---: | :---: | :---: | :---: |
| $1-200$ | 166 | 200 | 0.830 |
| $201-400$ | 100 | 200 | 0.500 |
| $401-700$ | 59 | 300 | 0.197 |
| $701-1000$ | 30 | 300 | 0.100 |
| $1001-1500$ | 5 | 500 | 0.010 |
| Frequency <br> densities <br> Graph | M1 |  |  |

NB Frequency densities can be scored on graph

(b) $\mathrm{Q}_{2}=200.5+\frac{(180-166)}{100} \times 200=\underline{228.5} \quad 228 / 229 / 230 \quad$ M1 A1
$\mathrm{Q}_{1}=0.5+\frac{90}{166} \times 200=\underline{108.933} \ldots \quad 109$ AWRT A1
$\mathrm{Q}_{3}=400.5+\frac{(270-266)}{59} \times 300=\underline{420.838} \quad$ AWRT 421/425 $\quad$ A1
( $\mathrm{n}=270.75 \Rightarrow \mathrm{Q}_{3}=424.6525$ )
$\mathrm{IQR}=420.830 \ldots-108.933 \ldots=\underline{311.905} \quad \mathrm{~B} 1 \mathrm{ft}$
B1ft 5
(c) $\Sigma \mathrm{fx}=110980$; $\Sigma \mathrm{fx} \mathrm{x}^{2}=58105890$

M1
Attempt at $\Sigma f x$ or $\Sigma f y$
$\Sigma \mathrm{f} y=748 ; \Sigma \mathrm{f} y^{2}=3943.5$ where $y=\frac{x-100.5}{100}$
M1
Attempt at $\Sigma \mathrm{Fx}^{2}$ or $\Sigma \int y^{2}$
$\mu=308.2777$
M1 A1 6
308 AWRT
$\sigma=257.6238$
258 AWRT
No working shown: SR B1 B1 only for $\mu, \sigma$.
(d) Median \& IQR
Sensible reason e.g. Assuming other years are skewed.

B1
B1 dep 2
29. (a) $\Sigma x=12075 ; \quad \Sigma x^{2}=15499685$
$\therefore \bar{x}=\frac{12075}{15}=\underline{805}$
B1
cao
sd $=\sqrt{\frac{15499685}{15}-805^{2}}=620.71491$
M1
$\sqrt{ } \&$ correct method
3 s.f. 621
A1 3
(NB Using $n-1$ gives 642.50125...) (643)
(b) $99,169,299,350,475,485,550,650,689,830$,

999, 1015, 1050, 2100, 2315
Attempt to order
$\therefore Q_{2}=\underline{650}$
A1
cao 650
$\therefore I Q R=Q_{3}-Q_{1}=1015-350=\underline{665}$
Attempt at $Q_{3}-Q_{1} \quad$ M1
cao 665 A1 4
(c) $Q_{3}+1.5\left(Q_{3}-Q_{1}\right)=1015+1.5 \times 665=2012.5$

Use of given outlier formula
$Q_{1}-1.5\left(Q_{3}-Q_{1}\right)=350-1.5 \times 665<0$ M1

Evidence both ends considered
$\therefore 2100$ and 2315 are outliers
A1 3


Two boxplots
B1
same scale
both labelled
Website
B1
Shop Box-plot B1

Both outliers
B1 4
NB: For shop, right band whisker drawn to 2012.5 is acceptable.
(e) Median website $>$ median shop

Website negative skew; shop approx symmetrical Ignoring outliers

Ranges approximately equal
Shop $\mathrm{Q}_{3}<$ Website $\mathrm{Q}_{3} \Rightarrow$ shop sales low value
Website sales more variable in value
$\mathrm{IQR}_{\mathrm{W}} \geq \mathrm{IQR}_{\mathrm{S}}$
Any two sensible comments B1 B1 2
30. Frequency densities: 3.0, 20.0, 9.0, 22.0, 3.0, 3.25

| Can be implied | M1 |
| :--- | ---: |
| from graph | A1 |


31. (a) $\bar{x}=\frac{20+15+\ldots+17}{14}=\frac{312}{14}=22.2857 \ldots$ (awrt 22.3)

M1 A1 2
(b)

| Bags of crisps |  | 1/0 means 10 | Total |
| :---: | :---: | :---: | :---: |
| 0 | 5 |  | (1) |
| 1 | $\begin{array}{lllll}0 & 1 & 3 & 5\end{array}$ | 7 | (5) |
| 2 | 0 0 05 |  | (3) |
| 3 | $0 \begin{array}{lll}0 & 1\end{array}$ |  | (3) |
| 4 | 02 |  | (2) |
| Label \& key <br> 2 correct rows <br> All correct |  |  |  |

(c) $Q_{2}=20 ; Q_{1}=13 ; Q_{3}=31$

B1; B1; B1 3
(d) $1.5 \times \mathrm{IQR}=1.5 \times(31-13)=27$ (can be implied) $31+27=58 ; 13-27=-14$ (both)
No outliers
(e)

(f) $\quad Q_{2}-Q_{1}=7 ; Q_{3}-Q_{2}=11 ; Q_{3}-Q_{2}>Q_{2}-Q_{1}$

Positive skew
A1 2
32. Frequency densities: $0.16,1.0,1.0,0.4,0.4,0.08$

M1, A1
Histogram: Scale and labels B1

Correct histogram
33. (a) $Q_{2}=\frac{16+16}{2}=16 ; Q_{1}=15 ; Q_{3}=16.5 ; \mathrm{IQR}=1.5 \quad \mathrm{M} 1 \mathrm{~A} 1 ; \mathrm{B} 1 ; \mathrm{B} 1 ; \mathrm{B} 1 \quad 5$
(b) $1.5 \times \mathrm{IQR}=1.5 \times 1.5=2.25$
$Q_{1}-1.5 \times \mathrm{IQR}=12.75 \Rightarrow$ no outliers below $Q_{1}$
$Q_{3}+1.5 \times \mathrm{IQR}=18.75 \Rightarrow 25$ is an outlier
A1
Boxplot, label scale
M1
14, 15, 16, 16.5, 18.75 (18)
Outlier
A1
A1 7
(c) $\bar{x}=\frac{322}{20}=16.1$

M1 A1 2
(d) Almost symmetrical/slight negative skew
Mean $(16.1) \approx$ Median $(16)$ and $Q_{3}-Q_{2}(0.5) \approx Q_{2}-Q_{1}(1.0)$

B1
B1 2
[16]
34. (a) Mode $=23$

B1 1
For $Q_{1}: \frac{n}{4}=10.5 \Rightarrow$ 11th observation $\quad \therefore Q_{1}=17$
B1

For $Q_{2}: \frac{n}{2}=21 \Rightarrow=\frac{1}{2}$ (21st $\left.\& 22 n d\right)$ observations
$\therefore Q_{2}=\frac{23+24}{2}=23.5$ M1 A1

For $Q_{3}: \frac{3 n}{4}=31.5 \Rightarrow$ 32nd observation $\quad \therefore Q_{3}=31 \quad$ B1 4
(c)


Box plot M1
Scale \& label M1
$Q_{1}, Q_{2}, Q_{3} \quad \mathrm{~A} 1$
11, 43
(d) From box plot or

$$
\begin{aligned}
& Q_{2}-Q_{1}=23.5-17=6.5 \\
& Q_{3}-Q_{2}=31-23.5=7.5 \text { (slight) positive skew }
\end{aligned}
$$

B1 1
(e) Back-to-back stem and leaf diagram

B1 1
35. (a) $\bar{y}=\frac{-467}{200} \quad$ (can be implied)

$$
\therefore \bar{x}=2.5 \bar{y}+755.0 \quad \text { M1 }
$$

$$
\begin{array}{rlr}
= & 2.5\left(\frac{-467}{200}\right)+755.0 & \text { A1 } \\
= & 749.1625 & \text { A } 1
\end{array}
$$

$$
\begin{array}{rlr}
S_{y}= & \sqrt{\frac{9179}{200}-\left(\frac{-467}{200}\right)^{2}} & \text { M1 A1 } \\
& =6.35946 & \text { A1 } \\
\therefore S_{x} & =2.5 \times 6.35946 & \text { M1 } \\
& =15.89865 \text { (accept awrt 15.9) } & \text { A1 }
\end{array}
$$

(b) Standard deviation $<\frac{2}{3}$ (interquartile range) ..... B1
Suggest using standard deviation since it shows less variation in the lifetimes ..... B1

1. Finding the midpoints of the given groups was predominantly carried out correctly with very few errors seen. In contrast, attempts at finding the width and height of the $26-30$ group were extremely varied, with most candidates finding this particularly challenging, especially in finding the height. In the majority of cases, candidates obtained the wrong width and height, mostly with no clear strategy, although these did multiply together to make 20.8 in some cases. Calculation of the mean was carried out successfully on the whole, although there were some apparent misconceptions, with quite a few candidates merely summing the midpoints (without multiplying by the frequency) and dividing this by 56.

The standard deviation proved to be more problematic, with frequent mistakes in both the formula and in their calculations. Some candidates used the sum of the $\mathrm{f}^{2} x$ 's and others the sum of the $(\mathrm{f} x)^{2}$ or the sum of the $\mathrm{f} x$ 's all squared in their formula. Very few candidates calculated $s$. Most candidates were able to use the correct interpolation technique to obtain the median, although many lost the accuracy mark through their use of 21 as the lower class boundary (which was relatively common) and /or 4 as the class width. Quite a few candidates worked with 28.5. A few candidates tried to apply the correct formula to the wrong class interval, however. Some candidates appeared to have a limited understanding of the class boundaries and failed to recognise the continuous nature of the data.

The majority of candidates were able to carry out a suitable test to determine the skewness of the data correctly. This mostly involved comparing $Q_{3}-Q_{2}$ to $Q_{2}-Q_{1}$ (with or without explicit substitutions), although the wrong conclusion was often drawn, either following on from a previous error in evaluating the median or from a lack of understanding of what their result was showing. A few students evaluated 3(mean - median)/standard deviation. Quite often the result of their test was described in words not figures, for example $Q_{2}$ is closer to $Q_{3}$ than $Q_{1}$. Some candidates merely attempted to describe the skewness without carrying out any test.
2. Parts (a) and (b) were answered very well although a few candidates gave the upper quartile as 39 or 39.5 (usually as a result of incorrectly rounding $\frac{3 n}{4}$ ) however the follow through marks meant that no further penalty need occur. A few found the upper and lower quartiles but failed to give the interquartile range. Most found the limit for an outlier using the given definition, although a few used $1.5 \times \mathrm{IQR}$, and went on to make a suitable comment about the one employee who needed retraining. There were some excellent box plots seen with all the correct features clearly present but a number failed to plot the outlier appropriately and simply drew their lower whisker to 7 . A not insignificant minority were confused by the absence of an upper whisker and felt the need to add one usually at $Q_{3}+\mathrm{IQR}$.
3. Part (a) gave most candidates two easy marks but the rest of the question proved more demanding. The calculation of the mean in part (b) was usually answered well but there were still some dividing by 8 and a few using $\frac{\sum f x^{2}}{\sum f x}$. The calculation of the standard deviation was better than on previous occasions with many reaching 0.421 but there is still some confusion over the formula with $\sqrt{\left(15889.5-\bar{x}^{2}\right)}$, a hybrid of the correct formula and $\mathrm{S}_{x x}$, being quite common. Candidates should be aware that the formula for standard deviation is very sensitive to rounding errors and an accurate value for the mean (stored on their calculator) should be used rather than a rounded answer. A number of candidates failed to use the given values for $\Sigma f x$ and $\Sigma f x^{2}$ and lost marks because of numerical slips. The attempts at interpolation in part (d) were
much improved with the correct fraction often being added to a lower class boundary. Unfortunately many used 2.95 or 2.5 as their class boundary and lost the marks. In part (e) the better candidates used their values for the mean and median and made an appropriate comment. Some spent the next page calculating $Q_{1}$ and $\mathrm{Q}_{3}$, often correctly, in order to use the quartiles to justify their description of the skewness.
4. There were many good answers to this question. The Venn diagram was often totally correct although a number failed to subtract for the intersections and obtained value of 35, 40 and 28 instead of 31, 36 and 24 for the numbers taking two options. Parts (b) and (c) were answered very well with only a minority of candidates failing to give probabilities. Part (d) proved straightforward for those who knew what was required but some attempted complicated calculations, often involving a product of probabilities, whilst others simply gave their answer as $4 / 180$.
5. Although there were more correct solutions than in previous papers for this type of question the process required to answer this question was not applied successfully by a large number of candidates. The most common error in part (a) was to give an answer of 0.8 . In tandem with this was an answer to part (b) of 7.5 where candidates recognised that the answer to part (a) times the answer to part (b) should be 6. Many candidates divided 9 by 3 in part (b) but failed to multiply by 2. Other candidates however produced two correct answers but nothing else. The variety of approaches may suggest some logical thinking rather than a taught approach to this type of problem.
6. Very few candidates got full marks for this question, being unable to perform the calculations for grouped data, although the mean caused the least problems. Those candidates with good presentation particularly those who tabulated their workings tended to fare better. In spite of the well defined groups many candidates subtracted or added 0.5 to the endpoints or adjusted the midpoints to be 0.5 less than the true value with the majority getting part (a) incorrect as a result. As usual all possible errors were seen for the calculation of $\Sigma \mathrm{fx}^{2}$ i.e. $(\Sigma \mathrm{fx})^{2}, \Sigma(\mathrm{fx})^{2}, \Sigma \mathrm{f}^{2} \mathrm{x}$ and $\Sigma \mathrm{x}^{2}$. Use of 17.1 for the mean in the calculation of the standard deviation led to the loss the accuracy mark. Candidates are once again reminded not to use rounded answers in subsequent calculations even though they usually gain full marks for the early answer. The comment in part (c) was often forgotten perhaps indicating that candidates are able to work out the figures but do not know what they mean, although many did appreciate in part (d) that there is no skew in a normal distribution. As opposed to question 1, correlation was often mentioned instead of skewness although again this is becoming less common.
7. This question was usually answered well. In part (b) some did not realise that they needed to check the lower limit as well in order to be sure that 110 was the only outlier. Part (c) was answered very well although some lost the last mark because there was no gap between the end of their whisker and the outlier. Part (d) was answered very well and most gave the correct values for $\sum y$ and $\sum y^{2}$ in the appropriate formula. A few tried to use the $\sum(y-\bar{y})^{2}$ approach but this requires all 10 terms to be seen for a complete "show that" and this was rare.

Part (e) was answered well although some gave the answer as -5.7 having forgotten the $10^{-3}$, or failed to interpret their calculator correctly. Many candidates gave comments about the
correlation being small or negative in part (f) but they did not give a clear reason for rejecting the parent's belief. Once again the interpretation of a calculated statistic caused difficulties.
8. Part (a) was not answered well. Many candidates attempted to calculate frequency densities but they often forgot to deal with the scale factor and the widths of the classes were frequently incorrect. There are a variety of different routes to a successful answer here but few candidates gave any explanation to accompany their working and it was therefore difficult for the examiners to give them much credit. The linear interpolation in part (b) was tackled with more success but a number missed the request for the Inter Quartile Range. Whilst the examiners did allow the use of $(n+1)$ here, candidates should remember that the data is being treated as continuous and it is therefore not appropriate to "round" up or down their point on the cumulative frequency axis. Although the mean was often found correctly the usual problems arose in part (c) with the standard deviation. Apart from those who rounded prematurely, some forgot the square root and others used $\sum f^{2} x$ or $\left(\sum f x\right)^{2}$ or $\frac{\sum f x^{2}}{\sum f x}$ instead of the correct first term in their expression and there was the usual crop of candidates who used $n=6$ instead of 104. The majority were able to propose and utilise a correct test for the skewness in part (d) with most preferring the quartiles rather than the mean and median. Few scored both marks in part (e) as, even if they chose the median, they missed mentioning the Inter Quartile Range. A number of candidates gave the mean and standard deviation without considering the implications of their previous result.
9. Most candidates were able to draw a six-branched tree diagram correctly, although a number of candidates had incorrect or missing labels. From a correct diagram most gained full marks in part (b). The conditional probability in part (c) once again caused difficulty for many of the candidates. Many of the responses in part (d) were, incorrectly, referring to the importance of testing people for a disease rather than referring to the probability in part (c).
10. Parts (a) to (d) represented a chance for all students who had an average grasp of statistics to score highly. The median in part (b) was incorrectly identified by a significant number of candidates, but the standard deviation was often correct.

Part (e) was done surprisingly well with students appearing to have a much greater understanding of what is required for a comparison than in previous years. Often numbers were stated without an actual comparison. Confusion was evident in some responses as skewness was often referred to as correlation. A small minority of candidates had failed to take note of the 'For the Balmoral Hotel' and had done some correct statistics for all 55 students.
11. A lot of fully correct Venn Diagrams were seen in part (a) although it was surprising the number who resorted to decimals rather than just using straightforward fractions; this often led to loss of many accuracy marks. A significant minority had negative numbers in their Venn diagram and saw nothing wrong in this when converting them to probabilities later in the question. Fewer candidates forgot the box this time. Part (c) proved to be the only difficult part, as many candidates struggled with the concept of conditional probability, and many denominators of 300 were seen.
12. The mean was calculated accurately by the majority of the candidates but the standard deviation calculation still caused problems for many. There were few summation errors but missing square roots or failing to square the mean were some of the more common errors.
Part (b) was poorly answered. The examiners were disappointed that such a sizeable minority failed to order the list and worked quite merrily with a median larger than their upper quartile. Some worked from the total of 2757 to get quartiles of $689.5,1378.5$ and 2067.5 whilst others used cumulative totals and obtained quartiles in the thousands but still failed to see the nonsensical nature of these figures. Those who did order the list used a variety of methods to try and establish the quartiles. Whilst the examiners showed some tolerance here any acceptable method should give a median of 198 but many candidates used 186 . Those who knew the rules usually scored full marks here and in parts (c) and (d).

The examiners followed through a wide range of answers in part (c) and most candidates were able to secure some marks for correctly identifying patients $B$ and $F$ and in part (d) for describing their skewness correctly.
13. The common error here was to assume that frequency equals the area under a bar, rather than using the relationship that the frequency is proportional to the area under the bar. Many candidates therefore ignored the statement in line 1 of the question about the histogram representing 140 runners and simply gave an answer of $12 \times 0.5=6$. A few candidates calculated the areas of the first 7 bars and subtracted this from 140, sadly they didn't think to look at the histogram and see if their answer seemed reasonable. Those who did find that the total area was 70 usually went on to score full marks. A small number of candidates had difficulty reading the scales on the graph and the examiners will endeavor to ensure that in any future questions of this type such difficulties are avoided. A small number of candidates had difficulty reading the scales on the graph and the examiners will endeavor to ensure that in any future questions of this type such difficulties are avoided.
14. Parts (a), (b) and (c) were generally well done, although in part (c) there were many with strange ideas of heavy instruments. In part (d) the majority of candidates were able to make a credible attempt at this with most giving one of the two possible solutions with a reason. The majority used the median and quartiles to find that the distribution was symmetrical. The use of the words 'symmetrical skew', similar to 'fair bias', is all too often seen but was accepted. Equal, even or normal skew were also often seen and were given no credit.
Part (e) was attempted successfully by a minority of candidates. A large number of candidates did not understand the distinction between z-values and probabilities. A lot gave 0.68 as $z$-value leading to the loss of the accuracy mark. Others tried to put various values into standard deviation formulae.
15. This was generally well answered. The majority of the errors occurred in part (c) by rounding too early and getting 18.4 for $a$. The regression line was often inaccurately plotted. In part (e) many used chocolate content to justify answer and often did not use the regression line to get a suitable price. Some misunderstood the question and attempted to find the best value in the second part of part (e)
16. Many candidates started well with this question, but a large number of inaccurate answers were seen for the latter parts. Part (a) was usually correct and part (b) was generally done well. In part (c) there were a lot of mistakes in finding midpoints and also $\Sigma f$. Most knew the correct method
for finding the mean, but rather fewer knew how to find the standard deviation in part (d) although most remembered to take the square root. Part (e) was very badly answered, with the majority unable to interpolate correctly which was often due to wrong class boundaries and / or class widths. In part (f), although the majority got an incorrect numerical value, most picked up the mark for interpreting their value correctly.
17. This question caused problems for many candidates. Part (a) did not always generate a comment about the skewness of the data and many who did eventually mention skewness thought it was negative. The calculation of the median in part (b) often caused difficulties. An endpoint of 19.5 was often used, but some thought the width was 9 not 10 and many simply opted for the midpoint of 24.5. The calculation of the mean in part (c) was sometimes the only mark scored by the weakest candidates and the examiners were disappointed at how many candidates were unable to find the standard deviation. Aside from the usual error of missing the square root or failing to square the mean, a number were using formulae such as $\sqrt{\frac{\sum f x^{2}}{\sum f x}}$. Most scored some marks in part (d) for attempting to use their values in the given formula, but the final mark required an answer accurate to 3 sf and this was rarely seen. In part (e) many failed to comment on the sign of their coefficient and there was often a discussion of correlation here rather than skewness. Of those who attempted the last two parts, part (g) was often successful, but in part (f), candidates often chose the mean because it used all the data rather than the median, which wouldn't be affected by the extreme values.
18. Parts (a) and (b) were not answered well. Few mentioned the type of variable in part (a) and in part (b) many simple stated that the frequency equals the area rather than stating that it was proportional to the area.

Many were able to give a correct calculation in part (c) but they sometimes failed to state that the 0.8 related to each individual child; the question was a "show that" and a final comment was required. The calculation in part (d) was usually correct.
19. Part (a) often scored full marks although some still mention 'mean' instead of 'median'. Part (d) was very straightforward for the vast majority of candidates. Those candidates who used a scale of 4 cm to 10 units were sometimes prone to placing the median inaccurately. Part (e) was also quite well done but some only listed the 5 important values with little or no mention of IQR, range, outliers or skewness. There was occasional confusion thinking the bigger numbers meant school B had done better.
20. In part (a) there were very few correct solutions. It was rare for a candidate to appreciate that the selection was without replacement. The rest of this question was well answered by many, although a surprising number averaged the two means in part (c).
21. This question was a good source of marks with most candidates able to find the correct values for the mode and median, but too many getting the upper quartile wrong. A surprising number of candidates had problems with finding the standard deviation. In quite a few cases the square
root was omitted but more often marks were lost due to the misinterpretation and misuse of standard formulae. This was not helped by some candidates ignoring given totals and calculating their own. In part (e) a large number of responses gave one reason rather than two.
22. It was very disappointing that so few candidates could carry out a simple analysis of a set of data. Few scored well.
(a) Relatively few candidates were able to state, "distance is a continuous variable." The most common wrong answer in this part referred to the unequal class widths.
(b) In general frequency densities were well done. The most common mistake was to calculate the incorrect class width, taking the first class width as 4 . Other mistakes were class width divided by frequency or frequency multiplied by class width although these were less prevalent than in previous examinations.
(c) Interpolation was not familiar to many candidates. Those pupils who did attempt to interpolate to find the median and quartiles were on the whole successful, common errors being the use of 50 instead of 50.5 or the wrong class interval. Many used the mid-point of the class for the quartiles or more frequently used $134 / 2$ or $(134+1) / 2$ as their responses for an estimate of the median.
(d) The mean was calculated successfully by the vast majority of candidates with only occasional error through using the sum of $\mathrm{f} x^{2}$ as opposed to the sum of $\mathrm{f} x$. The standard deviation proved more difficult - where students used the wrong formula, omitted the square root or lost accuracy marks through using the rounded value of the mean. Some candidates wasted time by recalculating the values given.
(e) Those candidates with sensible values for their quartiles managed to substitute successfully to calculate the coefficient although it was surprising how many could not get 0.14 from a correct expression. On the whole they drew the correct conclusion about the data being positively skewed, although a small number of candidates managed a correct calculation and then concluded negative skewness.
(f) Although a fair number of students could give a reason to confirm that the skewness was positive, most lost this mark by not justifying their comment using numerical values.
23. (a) The vast majority of candidates were able to make an attempt at drawing a box plot though labels were not always added and the upper whisker often extended to 63. For many candidates this was the only mark they obtained for the question. Few candidates bothered, or were able, to use the information regarding 1.5 IQR in order to identify the limits of acceptable data. Of those candidates who did show some working more often than not, they did not do so in enough detail. The number 24 was usually implied but candidates often ignored showing working for the lower end. The numbers of 52 and -12 were visible on a number of papers, but the conclusion about which numbers were outliers was often omitted.
(b) The majority of candidates recognized positive skewness, but many did not justify their answer numerically. Some candidates did not understand the request about the "distribution of delays" and gave an interpretation more suited to part (c).
(c) Most managed a comment on the distribution that was relevant, but few wrote in terms of whether passengers would be bothered by the delays - the majority of students used technical statistical terms, referring to quartiles and percentages of the data, rather than simply interpreting the data in non-technical language.
24. A lack of detailed labelling in the box plot was common. Candidates should realise that 3 marks for parts where they are comparing etc. requires them to find three relevant points. Many only had one or 2 points and seemed to think that if they wrote enough about one point they could get the 3 marks. The last part was not well interpreted by many. They were likely to just say that the 2 values for $\mathrm{Q}_{3}$ were the same. Most candidates can find quartiles and know how to display the information in box plots. There are still some candidates who do not draw a clearly labelled axis for their scale. Candidates need to remember that the purpose is to compare data so the scale needs to be the same for both sets of data. Some candidates can give good comparisons referring to range, IQR, median and quartiles, but many give vague descriptions concerning 'spread' and 'average' which gain no marks. They should be encouraged to be specific in their descriptions. Very few can interpret the upper quartile in context.
25. Many candidates were able to calculate the median and both quartiles accurately; the most common error was to give Keith’s median as 201. Some candidates ignored or failed to show the calculation of outliers. However, the vast majority of candidates were able to draw a reasonable box plot although the scale was often unlabelled. Other common errors were to extend the left hand whisker to 146 and the right hand whisker to 269 with 266 marked on as a bound for outliers.

In part (c), a failure to show working was all too common. Asif's distribution was often said to be "negatively skewed", with only a minority qualifying it as weak or almost symmetrical. Some candidates confused negative skew with positive skew.
26. Part (a) produced a poor response, with very few candidates realising when a histogram should be used. Part (b) was answered correctly by a very large majority of candidates. Most candidates appreciated the need for frequency densities and they were usually calculated accurately. The chosen scale was often good, but unsuitable scales are still seen too frequently at this level. Candidates generally labelled their axes. The heights if the rectangles were usually correctly plotted, but unsuitable scales sometimes proved a hindrance to candidates.
27. Candidates knew how to tackle this question but too many of them did not pay sufficient attention to detail. They often calculated the variance and not the standard deviation and their arithmetic was not always as accurate as it should have been. Some poor computational methods were seen when calculating the standard deviation. The stem and leaf diagram caused few problems for the candidates but few of them presented it in the most appropriate form. The mode and quartiles were often correct and it was pleasing to see many of them making a good attempt to compare and contrast the attendance data.
28. This was a long question that needed an understanding of several different but frequently linked concepts and many of the candidates did not have the stamina for such a question. Common errors included the drawing of a bar chart; fd = class width/frequency; poorly drawn histograms; quartiles calculated without using a lower class boundary; IQR omitted; a variety of incorrect mid-points; poor arithmetic and no appreciation of the skewness of the data. For a routine type of question the overall response was very disappointing.
29. This question posed few conceptual problems for the candidates but few of them gained full marks. The majority of lost marks were the result of candidates not paying sufficient attention to detail. The standard deviation was rarely given to an appropriate degree of accuracy and some candidates did not look for outliers at both ends of the data set. Although choosing a scale that would fit on the grid supplied for the candidates was not easy, most managed to do so but then forgot to label the axis or the two box plots. Others ignored the outliers even though they had identified them. Whilst candidates tried to find two sensible comparisons very few expressed them clearly.
30. There was still some uncertainty about how to calculate frequency densities. There were examples of candidates using (class width)/frequency or mid-point $\times$ frequency. Wrongly labelled axes, poor scales and wrong bases for the histogram bars lost marks for many candidates. A completely correct histogram was not very common.
31. Most candidates could make a reasonable attempt at this question. The stem and leaf diagram rarely had a label and although almost all candidates gave the correct value for the median, far too many did not give correct values for the other quartiles. Showing that there were no outliers was not always well attempted since many could not calculate the IQR correctly. The box plot was often spoiled by having no label, a poor or no proper scale and inaccurate plotting. Positive skewness was usually recognised and a correct justification was often given.
32. This question was generally well answered but some candidates could not work out frequency densities correctly. Many histograms were poorly labelled and many candidates gained marks on this question only because examiners followed through their frequency densities. There were still too many candidates who drew bar charts instead of histograms.
33. The upper quartile caused problems for many candidates but they should have seen an example with 20 observations and been able to deal with the quartiles. The values obtained by the candidates were followed through and this allowed many of them to score most of the marks. Working for the outliers was often omitted, with a loss of marks as stated in the rubric, and the label on the box plot was often missing. The mean was usually correct and some attempt to comment on the skewness was nearly always made.
34. No Report available for this question.
35. No Report available for this question.


[^0]:    "70 seen anywhere" B1
    Number of runners is 12

[^1]:    Freq / class width (implied)
    M1
    Scales and labels B1
    Histogram, no gaps \& their fd M1
    All correct

