

Recap 1

Algebraic Fractions

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(a) Show that $f(x) = \frac{x^2+x+1}{(x+2)^2}$, $x \neq -2$.

(4)

(b) Show that $x^2+x+1 > 0$ for all values of x .

(3)

(c) Show that $f(x) > 0$ for all values of x , $x \neq -2$.

(1)

$$a) \quad f(x) = \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2}$$

$$f(x) = \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2}$$

$$f(x) = \frac{x^2 + x + 1}{(x+2)^2}$$

$$b) \quad x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{4} \\ = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$$

$$c) \quad f(x) = \frac{x^2 + x + 1}{(x+2)^2} = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$$

Both numerator and denominator > 0 for $x \neq -2$

$\therefore f(x) > 0$ for all x except $x = -2$