

Proof

The square root of 2 is irrational

Assume $\sqrt{2}$ is rational

$$\therefore \sqrt{2} = \frac{P}{Q} \quad \text{where } P, Q \text{ are coprime integers}$$

$$2 = \frac{P^2}{Q^2}$$

$$2Q^2 = P^2$$

$$\Rightarrow P^2 \text{ is even}$$

$$\Rightarrow P \text{ is even} \quad \text{say } 2p$$

$$\sqrt{2} = \frac{2p}{Q}$$

$$2 = \frac{4p^2}{Q^2}$$

$$2Q^2 = 4p^2$$

$$Q^2 = 2p^2$$

$$\Rightarrow Q^2 \text{ is even}$$

$$\Rightarrow Q \text{ is even} \quad \text{say } 2q$$

$$\sqrt{2} = \frac{P}{Q} = \frac{2p}{2q}$$

This contradicts P, Q being coprime
 \therefore assumption $\sqrt{2}$ is rational was wrong
Conclusion $\sqrt{2}$ is irrational