# 6 Unit Specifications

## 6.1 INTRODUCTION TO ADVANCED MATHEMATICS, C1 (4751) AS

### Objectives

To build on and develop the techniques students have learnt at GCSE so that they acquire the fluency required for advanced work.

### Assessment

**Examination** (72 marks)
1 hour 30 minutes.
The examination paper has two sections.

Section A: 8-10 questions, each worth no more than 5 marks.
Section Total: 36 marks

Section B: three questions, each worth about 12 marks.
Section Total: 36 marks

### Assumed Knowledge

Candidates are expected to know the content of Intermediate Tier GCSE*.

*See note on page 34.

### Subject Criteria

The Units *C1* and *C2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The Units *C1*, *C2*, *C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

### Calculators

**No calculator is allowed in the examination for this module.**

In the MEI Structured Mathematics specification, graphical calculators are allowed in the examinations for all units except *C1*.

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

*Competence statements marked with an asterisk * are assumed knowledge and will not form the basis of any examination questions. These statements are included for clarity and completeness.*

## MATHEMATICAL PROCESSES

### Proof

*The construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language pervade the whole of mathematics at this level. These skills, and the Competence Statements below, are requirements of all the modules in these specifications.*

| | | |
|---|---|---|
| Mathematical argument | C1p1 | Understand and be able to use mathematical language, grammar and notation with precision. |
| | 2 | Be able to construct and present a mathematical argument. |

### Modelling

*Modelling pervades much of mathematics at this level and a basic understanding of the processes involved will be assumed in all modules*

### The Modelling Cycle

| | | |
|---|---|---|
| The modelling cycle. | C1p3 | Be able to recognise the essential elements in a modelling cycle. |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |
| C1p1 | Equals, does not equal, identically equals, therefore, because, implies, is implied by, necessary, sufficient | $=, \neq, \therefore,$ $\Rightarrow, \Leftarrow, \Leftrightarrow$ | |
| 2 | Construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language. | | |
| 3 | The elements are illustrated on the diagram in Section 5.2. | | |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| ALGEBRA | | |
|---|---|---|
| The basic language of algebra. | C1a1 | Know and be able to use vocabulary and notation appropriate to the subject at this level. |
| Solution of equations. | 2 | * Be able to solve linear equations in one unknown. |
| | 3 | Be able to change the subject of a formula. |
| | 4 | Know how to solve an equation graphically. |
| | 5 | Be able to solve quadratic equations. |
| | 6 | Be able to find the discriminant of a quadratic function and understand its significance. |
| | 7 | Know how to use the method of completing the square to find the line of symmetry and turning point of the graph of a quadratic function. |
| | 8 | * Be able to solve linear simultaneous equations in two unknowns. |
| | 9 | Be able to solve simultaneous equations in two unknowns where one equation is linear and one is of 2nd order. |
| | 10 | Know the significance of points of intersection of two graphs with relation to the solution of simultaneous equations. |
| Inequalities. | 11 | Be able to solve linear inequalities. |
| | 12 | Be able to solve quadratic inequalities. |
| Surds. | 13 | Be able to use and manipulate surds. |
| | 14 | Be able to rationalise the denominator of a surd. |
| Indices. | 15 | Understand and be able to use the laws of indices for all rational exponents. |
| | 16 | Understand the meaning of negative, fractional and zero indices. |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## ALGEBRA

| | | | |
|---|---|---|---|
| C1a1 | Expression, function, constant, variable, term, coefficient, index, linear, identity, equation. | $\mathrm{f}(x)$ | Formal treatment of functions. |
| 2 | Including those containing brackets and fractions. | | |
| 3 | Including cases where the new subject appears on both sides of the original formula, and cases involving squares and square roots. | | |
| 4 | Including repeated roots. | | |
| 5 | By factorising, completing the square, using the formula and graphically. | | |
| 6 | The condition for distinct real roots is: Discriminant > 0<br>The condition for repeated roots is: Discriminant = 0 | Discriminant $= b^2 - 4ac$. | Complex roots. |
| 7 | The graph of $y = a(x+p)^2 + q$ has a turning point at: $(-p, q)$ and a line of symmetry $x = -p$ | | |
| 8 | By elimination, substitution and graphically. | | |
| 9 | Analytical solution by substitution. | | |
| 10 | | | |
| 11 | Including those containing brackets and fractions. | | |
| 12 | Algebraic and graphical treatment of quadratic inequalities. | | Examples involving quadratics which cannot be factorised. |
| 13 | | | |
| 14 | e.g. $\frac{1}{5+\sqrt{3}} = \frac{5-\sqrt{3}}{22}$ | | |
| 15 | $x^a \times x^b = x^{a+b}, x^a \div x^b = x^{a-b}, (x^a)^n = x^{an}$ | | |
| 16 | $x^{-a} = \frac{1}{x^a}, x^{\frac{1}{a}} = \sqrt[a]{x}, \ x^0 = 1$ | | |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| COORDINATE GEOMETRY | | |
|---|---|---|
| The coordinate geometry of straight lines. | C1g1 | *Know the equation $y = mx + c$. |
| | 2 | Know how to specify a point in Cartesian coordinates in two dimensions. |
| | 3 | Know the relationship between the gradients of parallel lines and perpendicular lines. |
| | 4 | * Be able to calculate the distance between two points. |
| | 5 | * Be able to find the coordinates of the midpoint of a line segment joining two points. |
| | 6 | Be able to form the equation of a straight line. |
| | 7 | Be able to draw a line when given its equation. |
| | 8 | Be able to find the point of intersection of two lines. |
| The coordinate geometry of curves. | 9 | * Know how to plot a curve given its equation. |
| | 10 | Know how to find the point of intersection of a line and a curve. |
| | 11 | Know how the find the point(s) of intersection of two curves. |
| | 12 | Understand that the equation of a circle, centre (0, 0), radius $r$ is $x^2 + y^2 = r^2$ |
| | 13 | Understand that $(x-a)^2 + (y-b)^2 = r^2$ is the equation of a circle with centre $(a,b)$ and radius $r$. |
| | 14 | Know that:<br>– the angle in a semicircle is a right angle;<br>– the perpendicular from the centre of a circle to a chord bisects the chord;<br>– the tangent to a circle at a point is perpendicular to the radius through that point. |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## COORDINATE GEOMETRY

| | | | |
|---|---|---|---|
| C1g1 | | | |
| 2 | | | |
| 3 | For parallel lines $m_1 = m_2$.<br>For perpendicular lines $m_1 m_2 = -1$. | | |
| 4 | | | |
| 5 | | | |
| 6 | $y - y_1 = m(x - x_1)$, $ax + by + c = 0$,<br>$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$. | | |
| 7 | By using gradient and intercept as well as by plotting points. | | |
| 8 | By solution of simultaneous equations. | | |
| 9 | By making a table of values. | | |
| 10 | | | |
| 11 | | | Equations of order greater than 2. |
| 12 | | | |
| 13 | | | |
| 14 | These results may be used in the context of coordinate geometry. | | Formal proofs of these results. |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| POLYNOMIALS | | |
|---|---|---|
| Basic operations on polynomials. | C1f1 | Know how to add, subtract, multiply and divide polynomials. |
| The factor theorem. | 2 | Understand the factor theorem and know how to use it to factorise a polynomial. |
| | 3 | Know how to use the factor theorem to solve a polynomial equation. |
| | 4 | Know how to use the factor theorem to find an unknown coefficient. |
| The remainder theorem. | 5 | Understand the remainder theorem and know how to use it. |
| Graphs. | 6 | Know how to sketch the graphs of polynomial functions. |
| Binomial expansions. | 7 | Know how to use Pascal's triangle in the binomial expansion of $(a+x)^n$ where $n$ is a positive integer. |
| | 8 | Know the notations ${}^nC_r$ and $\binom{n}{r}$, and their relationship to Pascal's triangle. |
| | 9 | Know how to use ${}^nC_r$ in the binomial expansion of $(a+b)^n$ where $n$ is a positive integer. |

| CURVE SKETCHING | | |
|---|---|---|
| Vocabulary. | C1C1 | Understand the difference between sketching and plotting a curve. |
| Quadratic curves. | 2 | Know how to sketch a quadratic curve with its equation in completed square form. |
| Polynomial curves. | 3 | Know how to sketch the curve of a polynomial in factorised form. |
| Transformations. | 4 | Know how to sketch curves of the forms $y = \mathrm{f}(x) + a$ and $y = \mathrm{f}(x-b)$, given the curve of $y = \mathrm{f}(x)$. |

| INTRODUCTION TO ADVANCED MATHEMATICS, C1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## POLYNOMIALS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C1f1 | Expanding brackets and collecting like terms. Division by linear expressions only. | | Division by non-linear expressions. |
| 2 | $\mathrm{f}(a)=0 \Leftrightarrow (x-a)$ is a factor of $\mathrm{f}(x)$. | | |
| 3 | $\mathrm{f}(a)=0 \Rightarrow x=a$ is a root of $\mathrm{f}(x)=0$. | | Equations of degree > 4. |
| 4 | Use of factors to determine zeros<br>$(x-a)$ is a factor of $\mathrm{f}(x) \Rightarrow \mathrm{f}(a)=0$. | | |
| 5 | The remainder when $\mathrm{f}(x)$ is divided by $(x-a)$ is $\mathrm{f}(a)$. | | |
| 6 | By factorising. | | Functions of degree > 4. |
| 7 | | | |
| 8 | The meaning of the term factorial. | ${}^n\mathrm{C}_r = \binom{n}{r}$<br>$= \frac{n!}{r!(n-r)!}$<br>$n! = 1.2.3...n$<br>${}^nC_0 = {}^nC_n = 1$ | |
| 9 | | | |

## CURVE SKETCHING

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C1C1 | Where appropriate, candidates will be expected to identify where a curve crosses the coordinate axes (in cases where the points of intersection are known or easily found), and its behaviour for large numerical values of $x$. | | Asymptotes. |
| 2 | The curve $y=a(x+p)^2+q$ has a minimum at $(-p,q)$. | | |
| 3 | Including cases of repeated roots. | | Functions of degree > 4. |
| 4 | Vector notation may be used for a translation.<br>Including working with sketches of graphs where functions are not defined algebraically.<br>Other transformations are covered in C3f2-5. | $\begin{pmatrix} b \\ a \end{pmatrix}$ | |

## 6.2 CONCEPTS FOR ADVANCED MATHEMATICS, C2 (4752) AS

### Objectives

To introduce students to a number of topics which are fundamental to the advanced study of mathematics.

### Assessment

**Examination** (72 marks)
1 hour 30 minutes.
The examination paper has two sections.

Section A: 8-10 questions, each worth no more than 5 marks.
Section Total: 36 marks

Section B: three questions, each worth about 12 marks.
Section Total: 36 marks

### Assumed Knowledge

Candidates are expected to know the content of Intermediate Tier GCSE* and *C1*.

*See note on page 34.

### Subject Criteria

The Units *C1* and *C2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |
| **ALGEBRA** | | |
| Logarithms. | C2a1 | Understand the meaning of the word logarithm. |
| | 2 | Understand the laws of logarithms and how to apply them. |
| | 3 | Know the values of $\log_a a$ and $\log_a 1$. |
| | 4 | Know how to convert from an index to a logarithmic form and vice versa. |
| | 5 | Know the function $y = a^x$ and its graph. |
| | 6 | Be able to solve an equation of the form $a^x = b$. |
| | 7 | Know how to reduce the equations $y = ax^n$ and $y = ab^x$ to linear form and, using experimental data, to draw a graph to find values of $a, n$ and $a, b$. |
| **SEQUENCES AND SERIES** | | |
| Definitions of sequences. | C2s1 | Know what a sequence of numbers is and the meaning of finite and infinite sequences. |
| | 2 | Know that a sequence can be generated using a formula for the $k^{\text{th}}$ term, or a recurrence relation of the form $a_{k+1} = \mathrm{f}(a_k)$. |
| | 3 | Know what a series is. |
| | 4 | Be familiar with $\sum$ notation. |
| | 5 | Know and be able to recognise the periodicity of sequences. |
| | 6 | Know the difference between convergent and divergent sequences. |
| Arithmetic series. | 7 | Know what is meant by arithmetic series and sequences. |
| | 8 | Be able to use the standard formulae associated with arithmetic series and sequences. |
| Geometric series. | 9 | Know what is meant by geometric series and sequences. |
| | 10 | Be able to use the standard formulae associated with geometric series and sequences. |
| | 11 | Know the condition for a geometric series to be convergent and be able to find its sum to infinity. |
| | 12 | Be able to solve problems involving arithmetic and geometric series and sequences. |

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## ALGEBRA

| | | | |
|---|---|---|---|
| C2a1 | $y = \log_a x \Leftrightarrow a^y = x$ | | |
| 2 | $\log_a(xy) = \log_a x + \log_a y$<br>$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$<br>$\log_a(x^k) = k\log_a x$ | | Change of base of logarithms. |
| 3 | $\log_a a = 1$, $\log_a 1 = 0$ | | |
| 4 | $x = a^n \Leftrightarrow n = \log_a x$ | | |
| 5 | For $a \geq 1$. | | |
| 6 | By taking logarithms of both sides. | | |
| 7 | By taking logarithms of both sides and comparing with the equation $y = mx + c$. | | |

## SEQUENCES AND SERIES

| | | | |
|---|---|---|---|
| C2s1 | | | |
| 2 | e.g. $a_k = 2 + 3k$; $a_{k+1} = a_k + 3$ with $a_1 = 5$. | $k^{\text{th}}$ term: $a_k$ | |
| 3 | With reference to the corresponding sequence. | | |
| 4 | Including the sum of the first $n$ natural numbers. | | |
| 5 | | | |
| 6 | e.g. convergent sequence $a_k = 3 - \frac{1}{k}$<br>e.g. divergent sequence $a_k = 1 + 2k^2$ | | Formal tests for convergence. |
| 7 | The term arithmetic progression (AP) may also be used. | 1st term, $a$<br>Last term, $l$<br>Common difference, $d$. | |
| 8 | The $n$th term, the sum to $n$ terms. | | |
| 9 | The term geometric progression (GP) may also be used. | 1st term, $a$<br>Common ratio, $r$. | |
| 10 | The $n$th term, the sum to $n$ terms. | $S_n$ | |
| 11 | Candidates will be expected to be familiar with the modulus sign in the condition for convergence. | $S_\infty = \frac{a}{1-r}$, $\lvert r \rvert < 1$ | |
| 12 | These may involve the solution of quadratic and simultaneous equations. | | |

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| TRIGONOMETRY | | |
|---|---|---|
| Basic trigonometry. | C2t1 | * Know how to solve right-angled triangles using trigonometry. |
| The sine, cosine and tangent functions. | 2 | Be able to use the definitions of sin $\theta$ and cos $\theta$ for any angle. |
| | 3 | Know the graphs of sin $\theta$, cos $\theta$ and tan $\theta$ for all values of $\theta$, their symmetries and periodicities. |
| | 4 | Know the values of sin $\theta$, cos $\theta$ and tan $\theta$ when $\theta$ is 0°, 30°, 45°, 60°, 90° and 180°. |
| Identities. | 5 | Be able to use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (for any angle). |
| | 6 | Be able to use the identity $\sin^2\theta + \cos^2\theta = 1$. |
| | 7 | Be able to solve simple trigonometric equations in given intervals. |
| Area of a triangle. | 8 | Know and be able to use the fact that the area of a triangle is given by $\frac{1}{2}ab\sin C$. |
| The sine and cosine rules. | 9 | Know and be able to use the sine and cosine rules. |
| Radians. | 10 | Understand the definition of a radian and be able to convert between radians and degrees. |
| | 11 | Know and be able to find the arc length and area of a sector of a circle, when the angle is given in radians. |

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## TRIGONOMETRY

| | | | |
|---|---|---|---|
| C2t1 | | | |
| 2 | e.g. by reference to the unit circle. | | |
| 3 | Their use to find angles outside the first quadrant. | | |
| 4 | Exact values may be expected. | | |
| 5 | e.g. solve $\sin\theta = 3\cos\theta$ for $0° \le \theta \le 360°$. | | |
| 6 | e.g. simple application to solution of equations. | | |
| 7 | e.g. $\sin\theta = 0.5 \Leftrightarrow \theta = 30°$, 150° in [0°, 360°]. | arcsin $x$<br>arccos $x$<br>arctan $x$ | Principal values (see C4)<br>General solutions. |
| 8 | | | |
| 9 | Use of bearings may be required. | | |
| 10 | | | |
| 11 | The results $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ where $\theta$ is measured in radians. | | |

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| CALCULUS | | |
|---|---|---|
| The basic process of differentiation. | C2c1 | Know that the gradient of a curve at a point is given by the gradient of the tangent at the point. |
| | 2 | Know that the gradient of the tangent is given by the limit of the gradient of a chord. |
| | 3 | Know that the gradient function $\frac{\mathrm{d}y}{\mathrm{d}x}$ gives the gradient of the curve and measures the rate of change of $y$ with respect to $x$. |
| Applications of differentiation to the graphs of functions. | 4 | Be able to differentiate $y = kx^n$ where $k$ is a constant, and the sum of such functions. |
| | 5 | Be able to find second derivatives. |
| | 6 | Be able to use differentiation to find stationary points on a curve: maxima, minima and points of inflection. |
| | 7 | Understand the terms increasing function and decreasing function. |
| | 8 | Be able to find the equation of a tangent and normal at any point on a curve. |
| Integration as the inverse of differentiation. | 9 | Know that integration is the inverse of differentiation. |
| | 10 | Be able to integrate functions of the form $kx^n$ where $k$ is a constant and $n \neq -1$, and the sum of such functions. |
| | 11 | Know what are meant by indefinite and definite integrals. |
| | 12 | Be able to evaluate definite integrals. |
| | 13 | Be able to find a constant of integration given relevant information. |
| Integration to find the area under a curve. | 14 | Know that the area under a graph can be found as the limit of a sum of areas of rectangles. |
| | 15 | Be able to use integration to find the area between a graph and the $x$-axis. |
| | 16 | Be able to find an approximate value of a definite integral using the trapezium rule, and comment sensibly on its accuracy. |

| CURVE SKETCHING | | |
|---|---|---|
| Stationary points. | C2C1 | Be able to use stationary points when curve sketching. |
| Stretches. | 2 | Know how to sketch curves of the form $y = a\mathrm{f}(x)$ and $y = \mathrm{f}(ax)$, given the curve of: $y = \mathrm{f}(x)$. |

| CONCEPTS FOR ADVANCED MATHEMATICS, C2 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

| **CALCULUS** | | | |
|---|---|---|---|
| C2c1 | | | |
| 2 | | $\frac{\mathrm{d}y}{\mathrm{d}x} = \text{Lim}_{\delta x \to 0} \frac{\delta y}{\delta x}$ | |
| 3 | The terms increasing function and decreasing function. | | |
| 4 | Simple cases of differentiation from first principles. Including rational values of *n*. | $\mathrm{f}'(x) = \text{Lim}_{h \to 0}\left(\frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}\right)$ | |
| 5 | | $\mathrm{f}''(x) = \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ | |
| 6 | | | |
| 7 | In relation to the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$. | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |
| 12 | e.g. $\int_1^3 (3x^2 + 5x - 1)\mathrm{d}x$. | | |
| 13 | e.g. Find *y* when $x = 2$ given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 5$ and $y = 7$ when $x = 1$. | | |
| 14 | General understanding only. | | Formal proof. |
| 15 | Includes areas of regions partly above and partly below the *x*-axis. | | |
| 16 | Comments on the error will be restricted to consideration of its direction and made with reference to the shape of the curve. | | Repeated applications of the trapezium rule (see C4). |

| **CURVE SKETCHING** | | | |
|---|---|---|---|
| C2C1 | Including distinguishing between them. | | |
| 2 | Simple cases only e.g. Given $\mathrm{f}(x) = \sin x$, sketch $y = \sin(2x)$ or $y = 3\sin x$. | | Combined transformations (see C3f2). |

## 6.3 METHODS FOR ADVANCED MATHEMATICS, C3 (4753) A2

### Objectives

To build on and develop the techniques students have learnt at AS Level, with particular emphasis on the calculus.

### Assessment

**Examination** (72 marks)
1 hour 30 minutes
The examination paper has two sections.

Section A: 5-7 questions, each worth at most 8 marks.
Section Total: 36 marks

Section B: two questions, each worth about 18 marks.
Section Total: 36 marks

**Coursework** (18 marks)

Candidates are required to undertake a piece of coursework on the numerical solution of equations (see pages 62 to 65).

### Assumed Knowledge

Candidates are expected to know the content for Units *C1* and *C2*.

### Subject Criteria

The Units *C1* and *C2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

| METHODS FOR ADVANCED MATHEMATICS, C3 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| PROOF | | |
|---|---|---|
| Methods of proof. | C3p1 | Understand, and be able to use, proof by direct argument, exhaustion and contradiction. |
| | 2 | Be able to disprove a conjecture by the use of a counter example. |

| EXPONENTIALS AND NATURAL LOGARITHMS | | |
|---|---|---|
| The exponential and natural logarithm. | C3a1 | Understand and be able to use the simple properties of exponential and logarithmic functions including the functions $\mathrm{e}^x$ and $\ln x$. |
| Functions. | 2 | Know the relationship between $\ln x$ and $\mathrm{e}^x$. |
| | 3 | Know the graphs of $y = \ln x$ and $y = \mathrm{e}^x$. |
| | 4 | Be able to solve problems involving exponential growth and decay. |

| FUNCTIONS | | |
|---|---|---|
| The language of functions. | C3f1 | Understand the definition of a function, and the associated language. |
| | 2 | Know the effect of combined transformations on a graph and be able to form the equation of the new graph. |
| | 3 | Be able, given the graph of $y = \mathrm{f}(x)$, to sketch related graphs. |
| | 4 | Be able to apply transformations to the basic trigonometrical functions. |
| | 5 | Know how to find a composite function, $\mathrm{gf}(x)$. |
| | 6 | Know the conditions necessary for the inverse of a function to exist and how to find it (algebraically and graphically). |
| | 7 | Understand the functions arcsin, arccos and arctan, their graphs and appropriate restricted domains. |
| | 8 | Understand what is meant by the terms odd, even and periodic functions and the symmetries associated with them. |
| The modulus function. | 9 | Understand the modulus function. |
| | 10 | Be able to solve simple inequalities containing a modulus sign. |

| METHODS FOR ADVANCED MATHEMATICS, C3 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## PROOF

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C3p1 | | | |
| 2 | | | |

## EXPONENTIALS AND NATURAL LOGARITHMS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C3a1 | | $\log_e x = \ln x$ | |
| 2 | Simplifying expressions involving exponentials and logarithms. | | |
| 3 | | | |
| 4 | | | |

## FUNCTIONS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C3f1 | Many-to-one, one-to-many, one-to-one, mapping, object, image, domain, codomain, range, odd, even, periodic. | | |
| 2 | Translation parallel to the *x*-axis.<br>Translation parallel to the *y*-axis.<br>Stretch parallel to the *x*-axis.<br>Stretch parallel to the *y*-axis.<br>Reflection in the *x*-axis.<br>Reflection in the *y*-axis.<br>Combinations of these transformations. | Translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$ | |
| 3 | $y = \mathrm{f}(x \pm a)$ $\quad y = \mathrm{f}(x) \pm a$<br>$y = \mathrm{f}(ax)$ $\quad y = a\mathrm{f}(x)$ for $a > 0$<br>$y = \mathrm{f}(-x)$ $\quad y = -\mathrm{f}(x)$. | | |
| 4 | Translations parallel to the *x*- and *y*-axes.<br>Stretches parallel to the *x*- and *y*-axes.<br>Reflections in the *x*- and *y*-axes. | | |
| 5 | | | |
| 6 | The use of reflection in the line $y = x$.<br>e.g. $\ln x$ ($x > 0$) is the inverse of $e^x$. | | |
| 7 | Their graphs and periodicity. | | |
| 8 | e.g. $x^n$ for integer values of *n*. | | |
| 9 | Graphs of linear functions involving a single modulus sign. | | |
| 10 | Including the use of inequalities of the form $\lvert x-a \rvert \le b$ to express upper and lower bounds, $a \pm b$, for the value of *x*. | | Inequalities involving more than one modulus sign. |

| METHODS FOR ADVANCED MATHEMATICS, C3 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| **CALCULUS** | | |
|---|---|---|
| The product, quotient and chain rules. | C3c1 | Be able to differentiate the product of two functions. |
| | 2 | Be able to differentiate the quotient of two functions. |
| | 3 | Be able to differentiate composite functions using the chain rule. |
| | 4 | Be able to find rates of change using the chain rule. |
| Inverse functions. | 5 | Be able to differentiate an inverse function. |
| Implicit differentiation. | 6 | Be able to differentiate a function defined implicitly. |
| Differentiation of further functions. | 7 | Be able to differentiate $e^{ax}$ and $\ln x$. |
| | 8 | Be able to differentiate the trigonometrical functions: $\sin x$; $\cos x$; $\tan x$. |
| Integration by substitution. | 9 | Be able to use integration by substitution in cases where the process is the reverse of the chain rule. |
| | 10 | Be able to use integration by substitution in other cases. |
| Integration of further functions. | 11 | Be able to integrate $\frac{1}{x}$. |
| | 12 | Be able to integrate $e^{ax}$. |
| | 13 | Be able to integrate $\sin x$ and $\cos x$. |
| Integration by parts. | 14 | Be able to use the method of integration by parts in cases where the process is the reverse of the product rule. |
| | 15 | Be able to apply integration by parts to $\ln x$. |

| METHODS FOR ADVANCED MATHEMATICS, C3 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

| CALCULUS | | | |
|---|---|---|---|
| C3c1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)}$ | | |
| 6 | e.g. $\ln y = 1 - x^2$. | | |
| 7 | | | |
| 8 | Including their sums and differences. | | |
| 9 | e.g. $(1+2x)^8$, $x(1+x^2)^8$, $xe^{x^2}$, $\frac{1}{2x+3}$<br>Where appropriate, recognition may replace substitution. | | |
| 10 | Simple cases only, e.g. $\frac{x}{2x+1}$. | | |
| 11 | | | |
| 12 | | | |
| 13 | | | Integrals involving arcsin, arccos and arctan forms. |
| 14 | e.g. $xe^x$. | | Integrals requiring more than one application of the method.<br>Products of $e^x$ and trigonometric functions. |
| 15 | | | |

| METHODS FOR ADVANCED MATHEMATICS, C3 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

## NUMERICAL METHODS

*This topic will not be assessed in the examination for C3, since it is the subject of the coursework.*

| | | |
|---|---|---|
| Change of sign. | C3e1 | Be able to locate the roots of $\mathrm{f}(x)=0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval of *x* in which $\mathrm{f}(x)$ is continuous. |
| | 2 | Be aware of circumstances under which change of sign methods may fail to give an expected root or may give a false root. |
| Fixed point iteration. | 3 | Be able to carry out a fixed point iteration after rearranging an equation into the form $x=\mathrm{g}(x)$. |
| | 4 | Understand that not all iterations converge to a particular root of an equation. |
| The Newton-Raphson method. | 5 | Be able to use the Newton-Raphson method to solve an equation. |
| Error Bounds. | 6 | Appreciate the need to establish error bounds when applying a numerical method. |
| Geometrical interpretation. | 7 | Be able to give a geometrical interpretation both of the processes involved and of their algebraic representation. |

## NUMERICAL METHODS

*This topic will not be assessed in the examination for C3, since it is the subject of the coursework.*

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| C3e1 | e.g. decimal search. | | |
| 2 | e.g. when the curve of $y = \mathrm{f}(x)$ touches the $x$-axis.<br>e.g. when the curve of $y = \mathrm{f}(x)$ has a vertical asymptote. | | |
| 3 | e.g. $x^3 - x - 4 = 0$ written as $x = \sqrt[3]{x+4}$ and so give rise to the iteration $x_{n+1} = \sqrt[3]{x_n + 4}$.<br>Staircase and cobweb diagrams. | | |
| 4 | The iteration $x_{n+1} = \mathrm{g}(x_n)$ converges to a root at $x = a$ if $\lvert \mathrm{g}'(a) \rvert < 1$ providing the iteration starts sufficiently close to $a$. | | Proof. |
| 5 | | | |
| 6 | Error bounds should be established within the numerical method and not by reference to an already known solution. | | |
| 7 | | | |

## Methods for Advanced Mathematics (C3) Coursework: Solution of Equations by Numerical Methods

### Rationale

The assessment of this unit includes a coursework task (Component 2) involving the solution of equations by three different numerical methods.

The aims of this coursework are that students should appreciate the principles of numerical methods and at the same time be provided with useful equation solving techniques.

The objectives are:

- that students should be able to solve equations efficiently, to any required level of accuracy, using numerical methods;
- that in doing so they will appreciate how to use appropriate technology, such as calculators and computers, as a mathematical tool and have an awareness of its limitations;
- that they show geometrical awareness of the processes involved.

This task represents 20% of the assessment and the work involved should be consistent with that figure, both in quality and level of sophistication.

Numerical methods should be seen as complementing analytical ones and not as providing alternative (and less accurate) ways of doing the same job. Thus, equations which have simple analytical solutions should not be selected. Accuracy should be established from within the numerical working and not by reference to an exact solution obtained analytically.

The intention of this piece of coursework is not merely to solve equations; students should be encouraged to treat it as an investigation and to choose their own equations.

### Requirements

**1** Students must solve equations by the following three methods:

- Systematic search for a change of sign using one of the methods: bisection; decimal search; linear interpolation. One root is to be found.
- Fixed point iteration using the Newton-Raphson method. The equation selected must have at least two roots and all roots are to be found.
- Fixed point iteration after rearranging the equation $\mathrm{f}(x) = 0$ into the form $x = \mathrm{g}(x)$. One root is to be found.

A different equation must be used for each method.

In addition, a student's write-up must meet the following requirements.

**2** One root of one of the equations must be found by all three methods. The methods used should then be compared in terms of their efficiency and ease of use.

**3** The write-up must include graphical illustrations of how the methods work on the student's equations.

**4** A student is expected to be able to give error bounds for the value of any root. This must be demonstrated in the case of the change of sign method (maximum possible error $0.5 \times 10^{-3}$), and for one of the roots found by the Newton-Raphson method (required accuracy five significant figures).

**5** For each method an example should be given of an equation where the method fails: that is, an expected root is not obtained, a root is not found or a false root is obtained. There must be an explanation, illustrated graphically, of why this happens. In this situation it is acceptable to use equations with known analytical solutions provided they are not trivial.

## Notation and Language

Students are expected to use correct notation and terminology. This includes distinguishing between the words function and equation, and between root and solution.

- For a *function* denoted by f($x$), the corresponding *equation* is f($x$) = 0. Thus the expression $x^3 - 3x^2 - 4x + 11$ is a function, $x^3 - 3x^2 - 4x + 11 = 0$ is an equation.
- The equation $x^3 - x = 0$ has three *roots*, namely $x$ = -1, $x$ = 0 and $x$ = +1. The *solution* of the equation is $x$ = -1, 0 or +1. Solving an equation involves finding all its roots.

## Trivial Equations

Students should avoid trivial equations both when solving them, and where demonstrating failure. For an equation to be non-trivial it must pass two tests.

**(i)** It should be an equation they would expect to work on rather than just write down the solution (if it exists); for instance $\frac{1}{(x-a)} = 0$ is definitely not acceptable; nor is any polynomial expressed as a product of linear factors.

**(ii)** Constructing a table of values for integer values of $x$ should not, in effect, solve the equation. Thus $x^3 - 6x^2 + 11x - 6 = 0$ (roots at $x$ = 1, 2 and 3) is not acceptable.

### Oral Communication

Each student must talk about the task; this may take the form of a class presentation, an interview with the assessor or ongoing discussion with the assessor while the work is in progress. Topics for discussion may include strategies used to find suitable equations and explanations, with reference to graphical illustrations, of how the numerical methods work.

### Use of Software

The use of existing computer or calculator software is encouraged, but students must:

- edit any print-outs and displays to include only what is relevant to the task in hand;
- demonstrate understanding of what the software has done, and how they could have performed the calculations themselves;
- appreciate that the use of such software allows them more time to spend on investigational work.

### Selection of Equations

Centres may provide students with a list of at least ten equations from which they can, if they wish, select those they are going to solve or use to demonstrate failure of a method. Such a list of equations should be forwarded to the Moderator with the sample of coursework requested. A new set of equations must be supplied with each examination series. Centres may, however, exercise the right not to issue a list, on the grounds that candidates stand to benefit from the mathematics they learn while finding their own equations.

## Methods for Advanced Mathematics (C3) Coursework: Assessment Sheet

**Task:** Candidates will investigate the solution of equations using the following three methods:

- Systematic search for change of sign using one of the three methods: decimal search, bisection or linear interpolation.
- Fixed point iteration using the Newton-Raphson method.
- Fixed point iteration after rearranging the equation $\mathrm{f}(x)=0$ into the form $x=\mathrm{g}(x)$.

<table>
<tr><td>Coursework Title</td><td colspan="5"></td></tr>
<tr><td>Candidate Name</td><td colspan="2"></td><td>Candidate Number</td><td colspan="2"></td></tr>
<tr><td>Centre Number</td><td colspan="2"></td><td>Date</td><td colspan="2"></td></tr>
<tr><th>Domain</th><th>Mark</th><th>Description</th><th colspan="2">Comment</th><th>Mark</th></tr>
<tr><td>Change of sign method (3)</td><td>1<br>1<br>1</td><td>The method is applied successfully to find one root of an equation.<br>Error bounds are stated and the method is illustrated graphically.<br>An example is given of an equation where one of the roots cannot be found by the chosen method. There is an illustrated explanation of why this is the case.</td><td colspan="2"></td><td></td></tr>
<tr><td>Newton-Raphson method (5)</td><td>1<br>1<br>1<br>1<br>1</td><td>The method is applied successfully to find one root of a second equation.<br>All the roots of the equation are found.<br>The method is illustrated graphically for one root.<br>Error bounds are established for one of the roots.<br>An example is given of an equation where this method fails to find a particular root despite a starting value close to it.<br>There is an illustrated explanation of why this has happened.</td><td colspan="2"></td><td></td></tr>
<tr><td>Rearranging f(x)=0 in the form x=g(x) (4)</td><td>1<br>1<br>1<br>1</td><td>A rearrangement is applied successfully to find a root of a third equation.<br>Convergence of this rearrangement to a root is demonstrated graphically and the magnitude of $\mathrm{g}'(x)$ is discussed.<br>A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root.<br>This failure is demonstrated graphically and the magnitude of $\mathrm{g}'(x)$ is discussed.</td><td colspan="2"></td><td></td></tr>
<tr><td>Comparison of methods (3)</td><td>1<br>1<br>1</td><td>One of the equations used above is selected and the other two methods are applied successfully to find the same root.<br>There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence.<br>There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software.</td><td colspan="2"></td><td></td></tr>
<tr><td>Written communication (1)</td><td>1</td><td>Correct notation and terminology are used.</td><td colspan="2"></td><td></td></tr>
<tr><td>Oral communication (2)</td><td>2</td><td>Presentation ☐<br>Interview ☐<br>Discussion ☐</td><td colspan="2">Please tick at least one box and give a brief report.</td><td></td></tr>
<tr><td colspan="3">Half marks may be awarded but the overall total must be an integer.<br>Please report overleaf on any help that the candidate has received beyond the guidelines</td><td colspan="2">TOTAL</td><td>18</td></tr>
</table>

**Coursework must be available for moderation by OCR**

## 6.4 APPLICATIONS OF ADVANCED MATHEMATICS, C4 (4754) A2

### Objectives

To develop the work in *C1*, *C2* and *C3* in directions which allow it to be applied to real world problems.

### Assessment

**Examination**

**Paper A:** (72 marks)
1 hour 30 minutes
The examination paper has two sections.

Section A: 5-7 questions, each worth at most 8 marks.
Section Total: 36 marks

Section B: two questions, each worth about 18 marks.
Section Total: 36 marks

**Paper B:** (18 marks)
1 hour

A comprehension task. (Further details on page 72.)
Total 18 marks

### Assumed Knowledge

Candidates are expected to know the content for *C1*, *C2* and *C3*.

### Subject Criteria

The Units *C1* and *C2* are required for Advanced Subsidiary GCE Mathematics in order to ensure coverage of the subject criteria.

The Units *C1*, *C2*, *C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

| APPLICATIONS OF ADVANCED MATHEMATICS, C4 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| **ALGEBRA** | | |
|---|---|---|
| The general binomial expansion. | C4a1 | Be able to form the binomial expansion of $(1+x)^n$ where $n$ is any rational number and find a particular term in it. |
| | 2 | Be able to write $(a+x)^n$ in the form $a^n\left(1+\frac{x}{a}\right)^n$ prior to expansion. |
| Rational expressions. | 3 | Be able to simplify rational expressions. |
| Partial fractions. | 4 | Be able to solve equations involving algebraic fractions. |
| | 5 | Know how to express algebraic fractions as partial fractions. |
| | 6 | Know how to use partial fractions with the binomial expansion to find the power series for an algebraic fraction. |
| **TRIGONOMETRY** | | |
| sec, cosec and cot. | C4t1 | Know the definitions of the sec, cosec and cot functions. |
| | 2 | Understand the relationship between the graphs of the sin, cos, tan, cosec, sec and cot functions. |
| | 3 | Know the relationships $\tan^2\theta+1=\sec^2\theta$ and $\cot^2\theta+1=\operatorname{cosec}^2\theta$. |
| Compound angle formulae. | 4 | Be able to use the identities for $\sin(\theta\pm\phi)$, $\cos(\theta\pm\phi)$, $\tan(\theta\pm\phi)$. |
| | 5 | Be able to use identities for $\sin 2\theta$, $\cos 2\theta$ (3 versions), $\tan 2\theta$. |
| Solution of trigonometrical equations. | 6 | Be able to solve simple trigonometrical equations within a given range including the use of any of the trigonometrical identities above. |
| | 7 | Know how to write the function $a\cos\theta\pm b\sin\theta$ in the forms $R\sin(\theta\pm\alpha)$ and $R\cos(\theta\pm\alpha)$ and how to use these to sketch the graph of the function, find its maximum and minimum values and to solve equations. |
| **PARAMETRIC EQUATIONS** | | |
| The use of parametric equations. | C4g1 | Understand the meaning of the terms parameter and parametric equations. |
| | 2 | Be able to find the equivalent cartesian equation for parametric equations. |
| | 3 | Recognise the parametric form of a circle. |
| | 4 | Be able to find the gradient at a point on a curve defined in terms of a parameter by differentiation. |

| APPLICATIONS OF ADVANCED MATHEMATICS, C4 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## ALGEBRA

| | | | |
|---|---|---|---|
| C4a1 | For $\lvert x \rvert < 1$ when $n$ is not a positive integer. | | |
| 2 | $\left\lvert \frac{x}{a} \right\rvert < 1$ when $n$ is not a positive integer. | | |
| 3 | Including factorising, cancelling and algebraic division. | | |
| 4 | | | |
| 5 | Proper fractions with the following denominators<br>$(ax+b)(cx+d)$<br>$(ax+b)(cx+d)^2$<br>$(ax+b)(x^2+c^2)$ | | Improper fractions. |
| 6 | | | |

## TRIGONOMETRY

| | | | |
|---|---|---|---|
| C4t1 | Including knowledge of the angles for which they are undefined. | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | Including identities from earlier units.<br>Knowledge of principal values. | | |
| 7 | | | |

## PARAMETRIC EQUATIONS

| | | | |
|---|---|---|---|
| C4g1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)}{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}$<br>Use to find the equations of tangents and normals to a curve. | | Stationary points. |

## APPLICATIONS OF ADVANCED MATHEMATICS, C4

| Specification | Ref. | Competence Statements |
|---|---|---|
| **CALCULUS** | | |
| Numerical integration. | C4c1 | Be able to use the trapezium rule to find an integral to a given level of accuracy. |
| Partial fractions. | 2 | Be able to use the method of partial fractions in integration. |
| Volumes of revolution. | 3 | Be able to calculate the volumes of the solids generated by rotating a plane region about the $x$-axis or the $y$-axis. |
| Differential equations. | 4 | Be able to formulate first order differential equations. |
| | 5 | Be able to solve first order differential equations. |
| **VECTORS** | | |
| Vectors in two and three dimensions. | C4v1 | Understand the language of vectors in two and three dimensions. |
| | 2 | Be able to add vectors, multiply a vector by a scalar, and express a vector as a combination of others. |
| The scalar product. | 3 | Know how to calculate the scalar product of two vectors, and be able to use it to find the angle between two vectors. |
| Coordinate geometry in two and three dimensions. | 4 | Be able to find the distance between two points, the midpoint and other points of simple division of a line. |
| The equations of lines and planes. | 5 | Be able to form and use the equation of a line. |
| | 6 | Be able to form and use the equation of a plane. |
| The intersection of a line and a plane. | 7 | Know that a vector which is perpendicular to a plane is perpendicular to any line in the plane. |
| | 8 | Know that the angle between two planes is the same as the angle between their normals. |
| | 9 | Be able to find the intersection of a line and a plane. |
| **COMPREHENSION** | | |
| The ability to read and comprehend a mathematical argument or an example of the application of mathematics. | C4p1 | Be able to follow mathematical arguments and descriptions of the solutions of problems when given in writing. |
| | 2 | Understand the modelling cycle and realise that it can be applied across many branches of mathematics. |

| APPLICATIONS OF ADVANCED MATHEMATICS, C4 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

| | | | |
|---|---|---|---|
| | **CALCULUS** | | |
| C4c1 | Use of increasing numbers of strips to improve the accuracy.<br>Use of increasing numbers of strips to estimate the error. | | Questions requiring more than 3 applications of the trapezium rule. |
| 2 | | | |
| 3 | | | Axes of rotation other than the $x$- and $y$-axes. |
| 4 | From given information about rates of change. | | |
| 5 | Differential equations with separable variables only. | | |
| | **VECTORS** | | |
| C4v1 | Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors. | $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, $\hat{\mathbf{r}}$<br>$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ | |
| 2 | Geometrical interpretation. | | |
| 3 | Including test for perpendicular vectors.<br>The angle between two lines. | $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$<br>$= \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ | |
| 4 | | | |
| 5 | In vector and cartesian form. | Line: $\mathbf{r} = \mathbf{a} + t\mathbf{u}$<br>$\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3} (= t)$. | |
| 6 | In vector and cartesian form. | Plane: $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$<br>$n_1x + n_2y + n_3z + d = 0$<br>where $d = -\mathbf{a.n}$ . | |
| 7 | If a vector is perpendicular to two non-parallel lines in a plane, it is perpendicular to the plane. | | |
| 8 | | | |
| 9 | | | |
| | **COMPREHENSION** | | |
| C4p1 | This may be assessed using a real world modelling context. | | |
| 2 | Abstraction from a real-world situation to a mathematical description; approximation simplification and solution; check against reality; progressive refinement. | | |

## Applications of Advanced Mathematics (C4) Comprehension Task

### Rationale

The aim of the comprehension task is to foster an appreciation among students that, in learning mathematics, they are acquiring skills which transcend the particular items of the specification content which have made up their course.

The objectives are that students should be able to:

- read and comprehend a mathematical argument or an example of the application of mathematics;
- respond to a synoptic piece of work covering ideas permeating their whole course;
- appreciate the relevance of particular techniques to real-world problems.

### Description and Conduct

Paper B of *Applications of Advanced Mathematics (C4)* consists of a comprehension task on which candidates are expected to take no more than 40 minutes. The task takes the form of a written article followed by questions designed to test how well candidates have understood it. Care will be taken in preparing the task to ensure that the language is readily accessible.

Candidates are allowed to bring standard English dictionaries into the examination. Full regulations can be found in the JCQ booklet *Instructions for conducting examinations*, published annually.

The use of bi-lingual translation dictionaries by candidates for whom English is not their first language has to be applied for under the access arrangements rules. Full details can be found in the JCQ booklet *Access Arrangements, Reasonable Adjustments and Special Consideration*, published annually.

### Content

By its nature, the content of the written piece of mathematics cannot be specified in the detail of the rest of the specification. However knowledge of GCSE and *C1*, *C2* and *C3* will be assumed, as well as the content of the rest of this unit. Candidates are expected to be aware of ideas concerning accuracy and errors. The written piece may follow a modelling cycle and in that case candidates will be expected to recognise it. No knowledge of mechanics will be assumed.

## 6.13 STATISTICS 1, S1 (4766) AS

### Objectives

To enable students to build on and extend the data handling and sampling techniques they have learnt at GCSE.

To enable students to apply theoretical knowledge to practical situations using simple probability models.

To give students insight into the ideas and techniques underlying hypothesis testing.

### Assessment

| | | |
|---|---|---|
| **Examination** | (72 marks)<br>1 hour 30 minutes<br>The examination paper has two sections: | |
| | Section A: | 5 - 7 questions, each worth at most 8 marks.<br>Section Total: 36 marks |
| | Section B: | two questions, each worth about 18 marks.<br>Section Total: 36 marks |

### Assumed Knowledge

Candidates are expected to know the content for Intermediate Tier GCSE*. In addition, they need to know the binomial expansion as covered in *C1*.

*See note on page 34.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

*The use of an asterisk * in a competence statement indicates assumed knowledge. These items will not be the focus of examination questions and are included for clarity and completeness. However, they may be used within questions on more advanced statistics.*

| STATISTICS 1, S1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| PROCESSES | | |
|---|---|---|
| *This section is fundamental to all the statistics units in this specification (Statistics 1-4). In this unit, the ideas may be used in examination questions but will not be their main subject.* | | |
| Statistical modelling. | S1p1 | Be able to abstract from a real world situation to a statistical description (model). |
| | 2 | Be able to apply an appropriate analysis to a statistical model. |
| | 3 | Be able to interpret and communicate results. |
| | 4 | Appreciate that a model may need to be progressively refined. |
| Sampling. | 5 | * Understand the meanings of the terms population and sample. |
| | 6 | * Be aware of the concept of random sampling. |
| **DATA PRESENTATION** | | |
| Classification and visual presentation of data. | S1D1 | * Know how to classify data as categorical, discrete or continuous. |
| | 2 | * Understand the meaning of and be able to construct frequency tables for ungrouped data and grouped data. |
| | 3 | * Know how to display categorical data using a pie chart or a bar chart. |
| | 4 | Know how to display discrete data using a vertical line chart. |
| | 5 | Know how to display continuous data using a histogram for both unequal and equal class intervals. |
| | 6 | * Know how to display and interpret data on a stem and leaf diagram. |
| | 7 | * Know how to display and interpret data on a box and whisker plot. |
| | 8 | Know how to display and interpret a cumulative frequency distribution. |
| | 9 | Know how to classify frequency distributions showing skewness. |

| STATISTICS 1, S1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## PROCESSES

*This section is fundamental to all the statistics units in this specification (Statistics 1-4).*
*In this unit, the ideas may be used in examination questions but will not be their main subject.*

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| S1p1 | Approximation and simplification involving appropriate distributions and probability models. | | Formal definitions. |
| 2 | | | |
| 3 | Their implications in real-world terms. | | |
| 4 | Check against reality. | | |
| 5 | | | |
| 6 | | | |

## DATA PRESENTATION

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| S1D1 | | | |
| 2 | Define class intervals and class boundaries. | | |
| 3 | | | |
| 4 | | | |
| 5 | Area proportional to frequency.<br>Use of the term frequency density will be expected. | | |
| 6 | The term stemplot is also widely used.<br>Stem and leaf diagrams will be expected to be sorted. | | |
| 7 | The term boxplot is also widely used.<br>The term outlier can be applied to data which are at least $1.5 \times$ IQR beyond the nearer quartile. | | |
| 8 | | | |
| 9 | Positive and negative skewness. | | Measures of skewness. |

| STATISTICS 1, S1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

| DATA PRESENTATION (continued) | | |
|---|---|---|
| Measures of central tendency and dispersion. | 10 | Know how to find median*, mean*, mode* and midrange. |
| | 11 | Know the usefulness of each of the above measures of central tendency. |
| | 12 | Know how to find range*, percentiles, quartiles* and interquartile range*. |
| | 13 | Know how to calculate and interpret mean squared deviation, root mean squared deviation, variance and standard deviation. |
| | 14 | Be able to use the statistical functions of a calculator to find mean, root mean square deviation and standard deviation. |
| | 15 | Know how the mean and standard deviation are affected by linear coding. |
| | 16 | Understand the term outlier. |

| STATISTICS 1, S1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

| DATA PRESENTATION | | | |
|---|---|---|---|
| 10 | For raw data, frequency distributions, grouped frequency distributions. | Mean = $\overline{x}$ | |
| 11 | | | |
| 12 | | | |
| 13 | For raw data, frequency distributions, grouped frequency distributions.<br>The term outlier can be applied to data which are at least 2 standard deviations from the mean.<br>$msd = \frac{S_{xx}}{n} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$ , $rmsd = \sqrt{msd}$ .<br>Sample variance: $s^2 = \frac{S_{xx}}{n-1} = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2$ . (✝)<br>Sample standard deviation: $s = \sqrt{\text{variance}}$ . (§) | *msd*, *rmsd*<br>$s^2$<br>$s$ | Corrections for class interval in these calculations. |
| 14 | | | |
| 15 | $y_i = a + bx_i \Rightarrow \overline{y} = a + b\overline{x},\ s_y{}^2 = b^2 s_x{}^2$ | | Proof of equivalence will not be tested. |
| 16 | The term outlier can be applied to data which are:<br>(a) at least 2 standard deviations from the mean;<br>(b) at least 1.5 × IQR beyond the nearer quartile. | | |

## DATA PRESENTATION

### Notation for sample variance and sample standard deviation

The notations $s^2$ and $s$ for sample variance and sample standard deviation, respectively, are written into both British Standards (BS3534-1, 1993) and International Standards (ISO 3534).

The definitions are those given above in equations (✝) and (§). The calculations are carried out using divisor $(n-1)$.

In this specification, the usage will be consistent with these definitions. Thus the meanings of 'sample variance', denoted by $s^2$, and 'sample standard deviation', denoted by $s$, are uniquely defined, as calculated with divisor $(n-1)$.

In early work in statistics it is common practice to introduce these concepts with divisor $n$ rather than $(n-1)$. However there is no recognised notation to denote the quantities so derived.

In this specification, in order to ensure unambiguity of meaning, these quantities will be referred to by the functional names of 'mean square deviation' and 'root mean square deviation'. The letters *msd* and *rmsd* will be used to denote their values.

Students should be aware of the variations in notation used by manufacturers on calculators and know what the symbols on their particular models represent.

| STATISTICS 1, S1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

## PROBABILITY

| Specification | Ref. | Competence Statements |
|---|---|---|
| Probability of events in a finite sample space. | S1u1 | Know how to calculate the probability of one event. |
| | 2 | Understand the concept of a complementary event and know that the probability of an event may be found by finding that of its complementary event. |
| Probability of two or more events which are: | 3 | Know how to draw sample space diagrams to help calculate probabilities. |
| | 4 | Know how to calculate the expected frequency of an event given its probability. |
| (i) mutually exclusive; | 5 | Understand the concepts of mutually exclusive events and independent events. |
| | 6 | Know to add probabilities for mutually exclusive events. |
| | 7 | Know to multiply probabilities for independent events. |
| | 8 | Know how to use tree diagrams to assist in the calculation of probabilities. |
| (ii) not mutually exclusive. | 9 | Know how to calculate probabilities for two events which are not mutually exclusive. |
| Conditional probability. | 10 | Be able to use Venn diagrams to help calculations of probabilities for up to three events. |
| | 11 | Know how to calculate conditional probabilities by formula, from tree diagrams or sample space diagrams |
| | 12 | Know that $P(B\|A) = P(B) \Leftrightarrow B$ and $A$ are independent. |

## DISCRETE RANDOM VARIABLES

| Specification | Ref. | Competence Statements |
|---|---|---|
| Probability distributions. | S1R1 | Be able to use probability functions, given algebraically or in tables. |
| Calculation of probability, expectation (mean) and variance. | 2 | Be able to calculate the numerical probabilities for a simple distribution. |
| | 3 | Be able to calculate the expectation (mean), $\mathrm{E}(X)$, in simple cases and understand its meaning. |
| | 4 | Be able to calculate the variance, $\mathrm{Var}(X)$, in simple cases. |

| STATISTICS 1, S1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

| PROBABILITY | | | |
|---|---|---|---|
| S1u1 | | | |
| 2 | | P(*A*)<br>$A'$ is the event 'Not *A*' | |
| 3 | | | |
| 4 | | Expected frequency:<br>*n* P(*A*) | |
| 5 | | | Formal notation and definitions. |
| 6 | To find P(*A* or *B*). | | |
| 7 | To find P(*A* and *B*)<br>Including the use of complementary events.<br>e.g. finding the probability of at least one 6 in five throws of a die. | | |
| 8 | | | |
| 9 | | | |
| 10 | Candidates should understand, though not necessarily in this form, the relation:<br>$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. | | Probability of a general or infinite number of events.<br>Formal proofs. |
| 11 | $\mathrm{P}(A \cap B) = \mathrm{P}(A).\mathrm{P}(B \mid A)$ | $\mathrm{P}(B \mid A)$ | |
| 12 | In this case $\mathrm{P}(A \cap B) = \mathrm{P}(A).\mathrm{P}(B)$. | | |

| DISCRETE RANDOM VARIABLES | | | |
|---|---|---|---|
| S1R1 | In *S1* questions will only be set on simple finite distributions. | | |
| 2 | | $\mathrm{P}(X = x)$ | |
| 3 | | $\mathrm{E}(X) = \mu$ | |
| 4 | Knowledge of $\mathrm{Var}(X) = \mathrm{E}(X^2) - \mu^2$. | $\mathrm{Var}(X) = \mathrm{E}[(X - \mu)^2]$ | |

| STATISTICS 1, S1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

## THE BINOMIAL DISTRIBUTION AND ITS USE IN HYPOTHESIS TESTING

| Specification | Ref. | Competence Statements |
|---|---|---|
| Situations leading to a binomial distribution. | S1H1 | Recognise situations which give rise to a binomial distribution. |
| | 2 | Be able to identify the binomial parameter $p$, the probability of success. |
| Calculations relating to binomial distribution. | 3 | Be able to calculate probabilities using the binomial distribution. |
| | 4 | Know that ${}^nC_r$ is the number of ways of selecting $r$ objects from $n$. |
| | 5 | Know that $n!$ is the number of ways of arranging $n$ objects in line. |
| Knowledge of mean. | 6 | Understand and apply mean = $np$. |
| Calculation of expected frequencies. | 7 | Be able to calculate the expected frequencies of the various possible outcomes from a series of binomial trials. |
| Hypothesis testing for a binomial probability $p$. | 8 | Understand the process of hypothesis testing and the associated vocabulary. |
| | 9 | Be able to identify Null and Alternative Hypotheses ($H_0$ and $H_1$) when setting up a hypothesis test on a binomial probability model. |
| | 10 | Be able to conduct hypothesis tests at various levels of significance. |
| | 11 | Be able to identify the critical and acceptance regions. |
| | 12 | Be able to draw a correct conclusion from the results of a hypothesis test on a binomial probability model. |
| | 13 | Understand when to apply 1- tail and 2- tail tests. |

| STATISTICS 1, S1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## THE BINOMIAL DISTRIBUTION AND ITS USE IN HYPOTHESIS TESTING

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| S1H1 | | | |
| 2 | As a model for observed data. | $B(n, p)$, $q = 1 - p$<br>$\sim$ means 'has the distribution'. | |
| 3 | Including use of tables of cumulative binomial probabilities. | | |
| 4 | | ${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{(n-r)!r!}$ | |
| 5 | | | |
| 6 | | | Formal proof of variance of the binomial distribution. |
| 7 | | | |
| 8 | Null hypothesis, alternative hypothesis.<br>Significance level, 1-tail test, 2-tail test.<br>Critical value, critical region, acceptance region. | | |
| 9 | | $H_0$, $H_1$ | |
| 10 | | | Normal approximation. |
| 11 | | | |
| 12 | | | |
| 13 | | | |

## 6.17 DECISION MATHEMATICS 1, D1 (4771) AS

### Objectives

To give students experience of modelling and of the use of algorithms in a variety of situations.

To develop modelling skills.

The problems presented are diverse and require flexibility of approach. Students are expected to consider the success of their modelling, and to appreciate the limitations of their solutions.

### Assessment

**Examination** (72 marks)
1 hour 30 minutes
The examination paper has two sections:

Section A: three questions, each worth 8 marks
Section Total: 24 marks

Section B: three questions each worth 16 marks
Section Total: 48 marks

### Assumed Knowledge

Candidates are expected to know the content of Intermediate Tier GCSE*.

*See note on page 34.

### Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

| DECISION MATHEMATICS 1, D1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |

## MODELLING

*The three units in Decision Mathematics are based on the use of the modelling cycle in solving problems*

| | | |
|---|---|---|
| The modelling cycle applied to real-world problems. | D1p1 | Be able to abstract from a real world problem to a mathematical model. |
| | 2 | Be able to analyse the model appropriately. |
| | 3 | Be able to interpret and communicate results. |
| | 4 | Be able progressively to refine a model as appropriate. |

## ALGORITHMS

| | | |
|---|---|---|
| Background and definition. | D1A1 | Be able to interpret and apply algorithms presented in a variety of formats. |
| | 2 | Be able to develop and adapt simple algorithms. |
| Basic ideas of complexity. | 3 | Understand the basic ideas of algorithmic complexity. |
| | 4 | Be able to analyse the complexity of some of the algorithms covered in this specification. |

## GRAPHS

| | | |
|---|---|---|
| Background and definitions. | D1g1 | Understand notation and terminology. |
| Use in problem solving. | 2 | Be able to model appropriate problems by using graphs. |

## NETWORKS

| | | |
|---|---|---|
| Definition. | D1N1 | Understand that a network is a graph with weighted arcs |
| Use in problem solving. | 2 | Be able to model appropriate problems by using networks |
| The minimum connector problem. | 3 | Know and be able to use Kruskal's and Prim's algorithms |
| The shortest path from a given node to other nodes. | 4 | Know and be able to apply Dijkstra's algorithm |

| DECISION MATHEMATICS 1, D1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## MODELLING

*The three units in Decision Mathematics are based on the use of the modelling cycle in solving problems*

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| D1p1 | Approximation and simplification. | | |
| 2 | Solution using an appropriate algorithm. | | |
| 3 | Implications in real world terms. | | |
| 4 | Check against reality; adapt standard algorithms. | | |

## ALGORITHMS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| D1A1 | Flowcharts; written English; pseudo-code. | | |
| 2 | To include sorting and packing algorithms.<br>Sorting: Bubble, Shuttle, insertion, quick sort.<br>Packing: Full-bin, first-fit, first-fit decreasing.<br>Candidates will be expected to know these packing algorithms. | | Candidates will not be required to memorise sorting algorithms. |
| 3 | Worst case; size of problem; that for quadratic algorithms doubling the size of a large problem can quadruple the solution time, etc. | Order notation, e.g. $O(n^2)$ for quadratic complexity. | |
| 4 | Kruskal; Prim (network and tabular forms); Dijkstra.<br>The requirements will also apply to algorithms in later modules (D2 and DC) at the stage when they are met. | | |

## GRAPHS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| D1g1 | Nodes/vertices; arcs/edges; trees; node order;<br>simple, complete, connected and bipartite graphs;<br>walks, trails, cycles and Hamilton cycles;<br>trees; digraphs; planarity. | Pictures (i.e. graphs), incidence matrices. | |
| 2 | e.g. Königsberg bridges; various river crossing problems; the tower of cubes problem; filing systems. | | |

## NETWORKS

| Ref. | Notes | Notation | Exclusions |
|---|---|---|---|
| D1N1 | | | |
| 2 | Use in modelling 'geographical' problems and other problems e.g. translating a book into different languages, e.g. the knapsack problem. | | |
| 3 | Kruskal's algorithm in graphical form only.<br>Prim's algorithm in graphical or tabular form. | | |
| 4 | | | |

| DECISION MATHEMATICS 1, D1 | | |
|---|---|---|
| **Specification** | **Ref.** | **Competence Statements** |
| | | **LINEAR PROGRAMMING** |
| Linear inequalities in two or more variables. | D1L1 | Be able to manipulate inequalities algebraically. |
| | 2 | Be able to illustrate linear inequalities in two variables graphically. |
| Formulation of constrained optimisation problems. | 3 | Be able to formulate simple maximisation of profit and minimisation of cost problems. |
| Solution of constrained optimisation problems. | 4 | Be able to use graphs to solve 2-D problems, including integer valued problems. |
| Algebraic interpretation of the graphical solution in 2 dimensions. | 5 | Be able to interpret solutions, including spare capacities. |
| | | **CRITICAL PATH ANALYSIS** |
| Using networks in project management. | D1X1 | Be able to construct and use a precedence network. |
| | 2 | Be able to construct and interpret a cascade chart. |
| | 3 | Be able to construct and interpret a resource histogram. |
| | 4 | Understand the use of alternative criteria in project optimisation. |
| | 5 | Be able to crash a network. |
| | | **SIMULATION** |
| Random variables. | D1Z1 | Know how to generate realisations of a discrete uniformly distributed random variable. |
| | 2 | Be able to use random variables to model discrete non-uniform random variables. |
| Simulation modelling. | 3 | Be able to build and use simple models. |
| | 4 | Be able to interpret results. |
| | 5 | Understand the need for repetition. |

| DECISION MATHEMATICS 1, D1 | | | |
|---|---|---|---|
| **Ref.** | **Notes** | **Notation** | **Exclusions** |

## LINEAR PROGRAMMING

| | | | |
|---|---|---|---|
| D1L1 | | | |
| 2 | | | Non-linear problems. |
| 3 | | Max $2x+3y$<br>s.t. $x+y\le 6$<br>$5x+2y\le 12$<br>$x\ge 0,\ y\ge 0$ | Non-linear problems |
| 4 | Showing alternating feasible points and their associated costs/profits. | | Solving problems in more than 2 dimensions. |
| 5 | | | |

## CRITICAL PATH ANALYSIS

| | | | |
|---|---|---|---|
| D1X1 | Including forward and backward passes, the identification of critical activities and the calculation of float (total and independent). | Activity on arc. | Knowledge of an algorithm for constructing a precedence network from a precedence table.<br>Knowledge of an algorithm for numbering activities.<br>Knowledge of an algorithm for resource smoothing. |
| 2 | | | |
| 3 | | | |
| 4 | Time; cost; use of resources. | | |
| 5 | Checking critical activities and for activities becoming critical. | | |

## SIMULATION

| | | | |
|---|---|---|---|
| D1Z1 | Drawing numbers from a hat; coins; dice; pseudo-random numbers from a calculator; simple pseudo-random number generators; random number tables. | | Continuous random variables. |
| 2 | Cumulative frequency methods, including rejecting values where necessary. | | |
| 3 | Hand simulations, including queuing situations. | | |
| 4 | | | |
| 5 | | | |