

12 A circle has equation $x^2 + y^2 - 8x - 4y = 9$.

(i) Show that the centre of this circle is C (4, 2) and find the radius of the circle. [3]

(ii) Show that the origin lies inside the circle. [2]

(iii) Show that AB is a diameter of the circle, where A has coordinates (2, 7) and B has coordinates (6, -3). [4]

(iv) Find the equation of the tangent to the circle at A. Give your answer in the form $y = mx + c$. [4]

i) $x^2 + y^2 - 8x - 4y = 9$

$$(x-4)^2 - 16 + (y-2)^2 - 4 = 9$$

$$(x-4)^2 + (y-2)^2 = 29$$

Circle centre (4, 2) radius $\sqrt{29}$

ii) Distance from origin to centre

$$= \sqrt{(4-0)^2 + (2-0)^2}$$

$$= \sqrt{16+4}$$

$$= \sqrt{20} < \sqrt{29}$$

\therefore inside circle

iii) $(x-4)^2 + (y-2)^2 = 29$ A(2, 7)
 B(6, -3)

A(2, 7) $(2-4)^2 + (7-2)^2 = 29$

$$4 + 25 = 29 \checkmark$$

$\therefore A$ on circle

$$\begin{aligned}
 B(6, -3) & \quad (6-4)^2 + (-3-2)^2 \\
 & = 2^2 + (-5)^2 \\
 & = 4 + 25 = 29 \quad \checkmark \\
 \therefore B \text{ on circle}
 \end{aligned}$$

$$\begin{aligned}
 \text{Distance } AB & = \sqrt{(6-2)^2 + (-3-7)^2} \\
 & = \sqrt{16 + 100} \\
 & = \sqrt{116} \\
 & = \sqrt{4 \times 29} \\
 & = 2\sqrt{29} = \text{twice radius}
 \end{aligned}$$

$\therefore AB$ is a diameter

IV Gradient from centre to A

$$\begin{array}{l}
 \text{Centre } (4, 2) \\
 A \quad (2, 7)
 \end{array}
 \quad \text{grad} = \frac{7-2}{2-4} = \frac{5}{-2} = -\frac{5}{2}$$

$$\therefore \text{grad of tgt} = +\frac{2}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{2}{5}(x - 2)$$

$$y - 7 = \frac{2}{5}x - \frac{4}{5}$$

$$y = \frac{2}{5}x + \frac{31}{5}$$

- 12 (i) Find the equation of the line passing through A (-1, 1) and B (3, 9). [3]
- (ii) Show that the equation of the perpendicular bisector of AB is $2y + x = 11$. [4]
- (iii) A circle has centre (5, 3), so that its equation is $(x - 5)^2 + (y - 3)^2 = k$. Given that the circle passes through A, show that $k = 40$. Show that the circle also passes through B. [2]
- (iv) Find the x -coordinates of the points where this circle crosses the x -axis. Give your answers in surd form. [3]

i)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}, \quad \frac{y-1}{9-1} = \frac{x-(-1)}{3-(-1)}$$

$$\frac{y-1}{8} = \frac{x+1}{4}$$

$$y-1 = \frac{8(x+1)}{4}$$

$$y-1 = 2(x+1)$$

$$y-1 = 2x+2$$

$$\underline{y = 2x+3}$$

ii)

$A(-1, 1)$	$B(3, 9)$
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Midpoint = $\left(\frac{-1+3}{2}, \frac{1+9}{2}\right)$

$$= (1, 5)$$

gradient of bisector will be $-\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{2}(x - 1)$$

$$y - 5 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{11}{2}$$

$$2y = -x + 11$$

$$\begin{array}{r} 2y + x = 11 \\ \hline \end{array}$$

iii) $(x-5)^2 + (y-3)^2 = k$

$A(-1,1)$ $(-1-5)^2 + (1-3)^2 = k$

$$(-6)^2 + (-2)^2 = k$$

$$36 + 4 = k$$

$$40 = k$$

$$\therefore (x-5)^2 + (y-3)^2 = 40$$

$B(3,9)$ $(3-5)^2 + (9-3)^2 = 40$

$$(-2)^2 + 6^2 = 40$$

$$4 + 36 = 40 \checkmark$$

$\therefore B$ on circle

iv) Cuts x-axis when $y=0$

$$(x-5)^2 + (0-3)^2 = 40$$

$$(x-5)^2 + 9 = 40$$

$$(x-5)^2 = 31$$

$$x-5 = \pm\sqrt{31}$$

$$x = 5 \pm \sqrt{31}$$

$$x = 5 + \sqrt{31} \quad x = 5 - \sqrt{31}$$
