10.

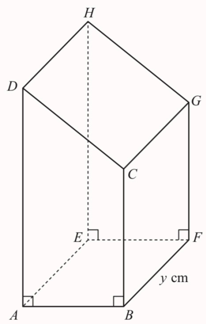

Figure 4

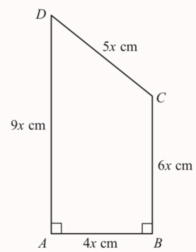

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

Area of Trapezium

$= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(9x+6x) \times 4x$

$= 30x^2$

Volume $30x^2y = 9600$

$x^2y = 320$

$y = \frac{320}{x^2}$

b) Surface Area

$5xy + 6xy + 4xy + 9xy + 30x^2 + 30x^2$

$S = 24xy + 60x^2$

$S = \frac{24 \times 320x}{x^2} + 60x^2$

$S = \frac{7680}{x} + 60x^2$

c) $S = 7680x^{-1} + 60x^2$

$\frac{dS}{dx} = -7680x^{-2} + 120x$

$\frac{dS}{dx} = -\frac{7680}{x^2} + 120x$

At st.pt $\frac{dS}{dx} = 0 \Rightarrow -\frac{7680}{x^2} + 120x = 0$

$-7680 + 120x^3 = 0$

$x^3 = \frac{7680}{120} = 64$

$x = \sqrt[3]{64} = 4$

Minimum Surface Area

$$= \frac{7680}{4} + 60 \times 4^2 = 2880 \text{ cm}^2$$

d) $$\frac{ds}{dx} = -7680x^{-2} + 120x$$

$$\frac{d^2s}{dx^2} = 2 \times 7680x^{-3} + 120$$

$$= \frac{15360}{x^3} + 120$$

when $x = 4$ $$= \frac{15360}{4^3} + 120$$

$$= 360 > 0 \quad \therefore \text{ a minimum}$$

8.

$V = \pi r^2 h$

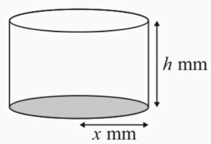

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x, **(1)**

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$ **(3)**

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum. **(5)**

(d) Calculate the minimum value of A, giving your answer to the nearest integer. **(2)**

(e) Show that this value of A is a minimum. **(2)**

a) $$60 = \pi x^2 h$$

$$h = \frac{60}{\pi x^2}$$

b) Surface Area

$$2\pi x h + \pi x^2 + \pi x^2$$

$$= \frac{120\pi x}{\pi x^2} + 2\pi x^2$$

$$= \frac{120}{x} + 2\pi x^2$$

c) $$A = 120x^{-1} + 2\pi x^2$$

$$\frac{dA}{dx} = -120x^{-2} + 2\pi x^2$$

At st. pt. $$\frac{dA}{dx} = 0 \Rightarrow -\frac{120}{x^2} + 2\pi x^2 = 0$$

$$-120 + 2\pi x^4 = 0$$

$$x^4 = \frac{120}{2\pi}$$

$$x = \sqrt[4]{\frac{60}{\pi}} = 2.09 \text{ mm}$$

d) Min $A = \frac{120}{2.09} + 2\pi \times 2.09^2$

$A = 84.86 = 85 \text{ mm}^2$ to nearest integer

e) $$\frac{dA}{dx} = -120x^{-2} + 2\pi x^2$$

$$\frac{d^2A}{dx^2} = 240x^{-3} + 4\pi x$$

$$= \frac{240}{x^3} + 4\pi x > 0$$ for all +ve x

$\therefore$ a minimum

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8.

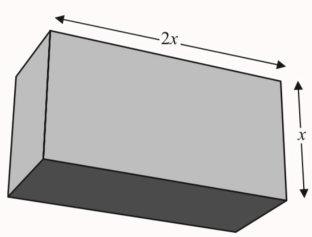

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)