13 A circle has equation $(x-5)^2 + (y-2)^2 = 20$.

(i) State the coordinates of the centre and the radius of this circle. **[2]**

(ii) State, with a reason, whether or not this circle intersects the y-axis. **[2]**

(iii) Find the equation of the line parallel to the line $y = 2x$ that passes through the centre of the circle. **[2]**

(iv) Show that the line $y = 2x + 2$ is a tangent to the circle. State the coordinates of the point of contact. **[5]**

i) Centre $(5, 2)$ Radius $\sqrt{20}$

ii) No, centre is 5 units to right of y-axis. Since radius $\sqrt{20} < 5$ the circle will not reach the y-axis.

iii) Line parallel to $y = 2x$ has gradient 2

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - 5)$$
$$y - 2 = 2x - 10$$
$$y = 2x - 8$$

iv) Solve $\begin{cases} (x-5)^2 + (y-2)^2 = 20 \\ y = 2x + 2 \end{cases}$

Sub for y in circle

$$(x-5)^2 + (2x+2-2)^2 = 20$$
$$(x-5)^2 + (2x)^2 = 20$$

$$x^2 - 10x + 25 + 4x^2 = 20$$

$$5x^2 - 10x + 5 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$\Rightarrow \quad x = 1$$

$$y = 2(1) + 2 = 4$$

Only one point of intersection so $y = 2x + 2$ is a tangent to circle, touching circle at $(1, 4)$

11

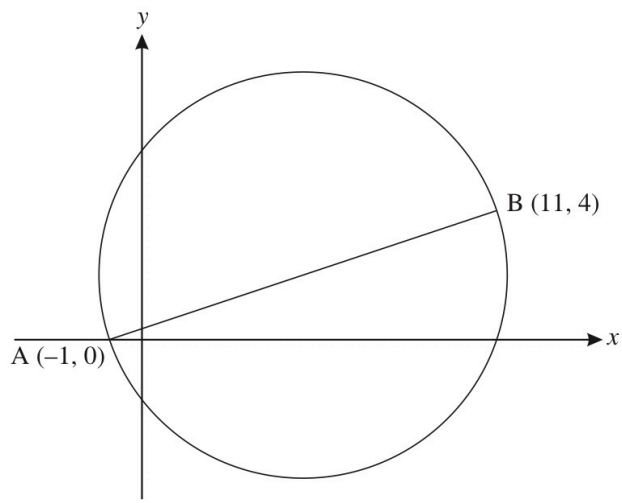

Fig. 11

Fig. 11 shows the points A and B, which have coordinates (–1, 0) and (11, 4) respectively.

(i) Show that the equation of the circle with AB as diameter may be written as

$$(x-5)^2 + (y-2)^2 = 40.$$ **[4]**

(ii) Find the coordinates of the points of intersection of this circle with the y-axis. Give your answer in the form $a \pm \sqrt{b}$. **[4]**

(iii) Find the equation of the tangent to the circle at B. Hence find the coordinates of the points of intersection of this tangent with the axes. **[6]**

i) Centre is mid-point of AB

$$= \left(\frac{-1+11}{2}, \frac{0+4}{2}\right) = (5, 2)$$

Radius is distance from centre to B

$$= \sqrt{(11-5)^2 + (4-2)^2}$$

$$= \sqrt{36+4}$$

$$= \sqrt{40}$$

$\therefore$ Eqn of circle is $(x-5)^2 + (y-2)^2 = 40$

ii) Intersects y-axis when $x = 0$

$$(0-5)^2 + (y-2)^2 = 40$$
$$(y-2)^2 = 15$$
$$y-2 = \pm\sqrt{15}$$
$$y = 2 \pm \sqrt{15}$$

Points of intersection

$$(0, 2+\sqrt{15}) \text{ and } (0, 2-\sqrt{15})$$

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iii) Gradient of diameter AB $= \frac{4-0}{11--1} = \frac{4}{12} = \frac{1}{3}$

Gradient of tangent at B $= -3$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -3(x - 11)$$
$$y - 4 = -3x + 33$$
$$y = -3x + 37$$

---

when $x = 0$, $y = 37$

when $y = 0$, $0 = -3x + 37$
$$3x = 37$$
$$x = \frac{37}{3}$$

Meets axes at

$(0, 37)$ and $\left(\frac{37}{3}, 0\right)$
