Exercise 6F

1) $U(-2, 8)$
$V(7, 7)$
$W(-3, -1)$

$\text{grad} = \frac{y_2 - y_1}{x_2 - x_1}$

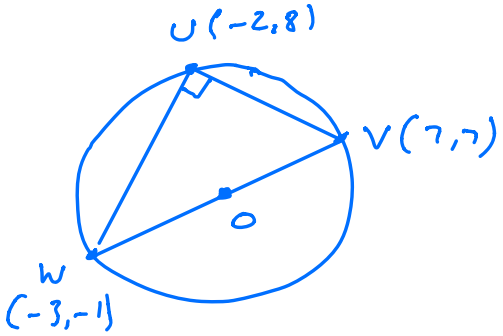
$\text{grad } UV = \frac{8-7}{-2-7} = \frac{1}{-9} = -\frac{1}{9}$

$\text{grad } VW = \frac{7--1}{7--3} = \frac{8}{10}$

$\text{grad } UW = \frac{8--1}{-2--3} = \frac{9}{1}$

$\text{grad } UV \times \text{grad } UW = -\frac{1}{9} \times \frac{9}{1} = -1$

$\therefore$ UV and UW are $\perp$
right angle at U



Centre is midpoint of VW

$\left(\frac{-3+7}{2}, \frac{-1+7}{2}\right)$

$= (2, 3)$

radius = OV

$= \sqrt{(7-2)^2 + (7-3)^2} = \sqrt{25 + 16} = \sqrt{41}$

Circle is $(x-2)^2 + (y-3)^2 = 41$

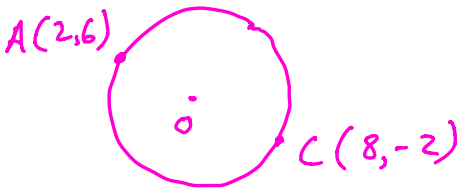
2) $A(2,6)$, $B(5,7)$, $C(8,-2)$

grad $AB = \frac{7-6}{5-2} = \frac{1}{3}$

grad $BC = \frac{7--2}{5-8} = \frac{9}{-3} = -3$

$\frac{1}{3} \times -3 = -1 \quad \therefore \perp$

right-angle at B so AC is a diameter
(angle in a semi-circle = 90°)



Centre $O = \left(\frac{2+8}{2}, \frac{6+-2}{2}\right)$

$= (5, 2)$

Radius $= OA$

$= \sqrt{(5-2)^2 + (2-6)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

Circle Eqn $(x-5)^2 + (y-2)^2 = 5^2$

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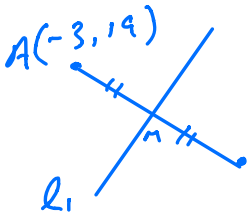
3) $A(-3,19)$, $B(9,11)$, $C(-15,1)$



$M = \left(\frac{-3+9}{2}, \frac{19+11}{2}\right)$

$M = (3, 15)$

grad $AB = \frac{19-11}{-3-9} = \frac{8}{-12} = -\frac{2}{3}$

grad of $l_1 = +\frac{3}{2}$

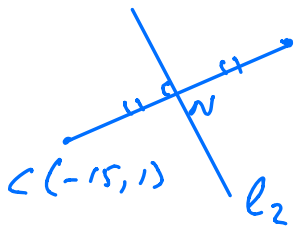
ℓ_1 $\quad y - y_1 = m(x - x_1)$

$y - 15 = \frac{3}{2}(x - 3)$

$2y - 30 = 3x - 9$

$\ell_1 \quad -21 = 3x - 2y$

ii Find $\perp$ bisector of AC



$N\left(\frac{-15 + -3}{2}, \frac{1 + 19}{2}\right)$

$N(-9, 10)$

grad AC $= \frac{19 - 1}{-3 - -15} = \frac{18}{12}$

$= \frac{3}{2}$

grad of $\ell_2 = -\frac{2}{3}$

$\ell_2 \quad y - y_1 = m(x - x_1)$

$y - 10 = -\frac{2}{3}(x - -9)$

$3y - 30 = -2(x + 9)$

$3y - 30 = -2x - 18$

$\ell_2 \quad 2x + 3y = 12$

$3x - 2y = -21 \quad (1)$

$2x + 3y = 12 \quad (2)$

(1) × 3 $\quad 9x - 6y = -63$ (3)

(2) × 2 $\quad 4x + 6y = 24$ (4)

(3) + (4) $\quad 13x = -39$

$x = \frac{-39}{13}$

$x = -3$

Sub for x in (2)

$2(-3) + 3y = 12$

$-6 + 3y = 12$

$3y = 18$

$y = 6$

Centre $= (-3, 6)$

Radius = B to centre

$= \sqrt{(9--3)^2 + (11-6)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

Circle $\quad (x+3)^2 + (y-6)^2 = 13^2$

Hwk Mixed Exercise 6 @ 1, 2, 3