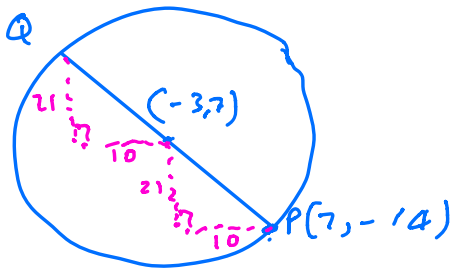
Exercise 6C Q12



$x^2+y^2+6x-14y=483$

$(x+3)^2-9+(y-7)^2-49=483$

$(x+3)^2+(y-7)^2=541$

centre $(-3,7)$

radius $\sqrt{541}$

$Q(-13,28)$

Exercise 6D

1) $(x-1)^2+(y-3)^2=45$

Meets x-axis when $y=0$

$(x-1)^2+(0-3)^2=45$

$(x-1)^2+9=45$

$(x-1)^2=36$

$x-1=\pm 6$

$x=\pm 6+1$

$x=7$ or $x=-5$

Meets x-axis at $(7,0)$ and $(-5,0)$

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3) $y=x+4$ $\qquad (x-3)^2+(y-5)^2=34$

Find points of intersection A and B

Sub for y in circle

$(x-3)^2+(x+4-5)^2=34$

$(x-3)^2+(x-1)^2=34$

$$x^2+9-6x+x^2+1-2x=34$$
$$2x^2-8x-24=0$$
$$x^2-4x-12=0$$
$$(x+2)(x-6)=0$$

Either $x+2=0$ or $x-6=0$

$x=-2$ $\quad$ $x=+6$

$y=-2+4$ $\quad$ $y=+6+4$

$y=+2$ $\quad$ $y=10$

$A(-2,2)$ $\quad$ $B(6,10)$

Q5 $\quad x-y-10=0$ $\quad$ $x^2-4x+y^2=21$

$x-10=y$ $\quad$ Sub for y in circle

$$x^2-4x+(x-10)^2=21$$
$$x^2-4x+x^2+100-20x=21$$
$$2x^2-24x+79=0$$

Discriminant b^2-4ac

$$=(-24)^2-4\times2\times79$$
$$=576-632$$
$$=-56<0$$

$\therefore$ no real roots

$\therefore$ no points of intersection

Exercise 6D $\quad$ Q 2, 4, 6, 8, 10, 12 $\quad$ FINISH FOR HOMEWORK

2) $(x-2)^2 + (y+3)^2 = 29$

Meets y-axis when $x=0$

$(0-2)^2 + (y+3)^2 = 29$

$4 + (y+3)^2 = 29$

$(y+3)^2 = 25$

$y+3 = \pm 5$

$y = \pm 5 - 3$

$y = 2$ or $y = -8$

Meets y-axis at $(0,2)$ and $(0,-8)$

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4) $x+y+5=0$ $\qquad$ $x^2+6x+y^2+10y-31=0$

$y=-x-5$ $\qquad$ Sub for y in circle

$x^2+6x+(-x-5)^2+10(-x-5)-31=0$

$x^2+6x+x^2+25+\cancel{10x}-\cancel{10x}-50-31=0$

$2x^2+6x-56=0$

$x^2+3x-28=0$

$(x+7)(x-4)=0$

$x=-7$ or $x=4$

$y=--7-5$ $\qquad$ $y=-4-5$

$y=2$ $\qquad$ $y=-9$

$(-7,2)$ and $(4,-9)$

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6) $x + y = 11$ $\qquad$ $x^2 + (y-3)^2 = 32$

$x = 11 - y$

Sub for x in circle

$(11-y)^2 + (y-3)^2 = 32$

$121 - 22y + y^2 + y^2 - 6y + 9 = 32$

$2y^2 - 28y + 98 = 0$

$y^2 - 14y + 49 = 0$

$(y-7)(y-7) = 0$

$\Rightarrow \left.\begin{array}{l} y = 7 \\ x = 4 \end{array}\right\}$ only one point of intersection $(4,7)$

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8) $x + y = a$ $\qquad$ $(x-p)^2 + (y-6)^2 = 20$

Meet at $(3,10)$ $\therefore$ $(3,10)$ is on both line and circle

$3 + 10 = a \quad \Rightarrow \quad \underline{a = 13}$

$(3-p)^2 + (10-6)^2 = 20$

$(3-p)^2 + 16 = 20$

$(3-p)^2 = 4$

$3 - p = \pm 2$

$3 \pm 2 = p$ $\qquad$ $p = 5$

or $p = 1$

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10) $y = kx \qquad x^2 - 10x + y^2 - 12y + 57 = 0$

Sub for y in circle

$$x^2 - 10x + k^2x^2 - 12kx + 57 = 0$$

$$(1+k^2)x^2 + (-10-12k)x + 57 = 0$$

For 2 distinct roots $b^2 - 4ac > 0$

$$(-10-12k)^2 - 4(1+k^2) \times 57 > 0$$

$$100 + 144k^2 + 240k - 228 - 228k^2 > 0$$

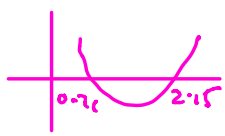
$$-84k^2 + 240k - 128 > 0$$

$$-21k^2 + 60k - 32 > 0$$

$$21k^2 - 60k + 32 < 0$$

$k = 2.15$

$k = 0.71$



$0.71 < k < 2.15$

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12) $y = 2x + 5 \qquad x^2 + kx + y^2 = 4$

Sub for y in circle

$$x^2 + kx + (2x+5)^2 = 4$$

$$x^2 + kx + 4x^2 + 20x + 25 = 4$$

$$5x^2 + (20+k)x + 21 = 0$$

For one point of intersection $b^2 - 4ac = 0$

$$(20+k)^2 - 4 \times 5 \times 21 = 0$$

$$(20+k)^2 = 420$$

$$20 + k = \pm\sqrt{420} = \pm 2\sqrt{105}$$

$$k = -20 \pm 2\sqrt{105}$$
