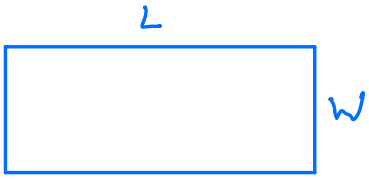
1) 200m of fencing - want rectangular pen
Maximise area of pen



Area $= L \times W$

Perimeter $= 2L + 2W = 200$

$L + W = 100$

$W = 100 - L$

Area $A = L(100 - L)$

$A = 100L - L^2$

$\frac{dA}{dL} = 100 - 2L$

For st. pt $\frac{dA}{dL} = 0$

$\Rightarrow 100 - 2L = 0$

$100 = 2L$

$50 = L$

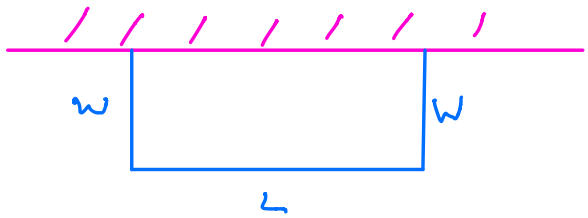
$\Rightarrow W = 50$

$\frac{d^2A}{dL^2} = -2 < 0$ $\therefore$ maximum

Max Area when $L = 50$, $W = 50$ (i.e. a square)

Max Area $= 50 \times 50 = 2500\text{m}^2$

2) Same sort of problem 400 m fence rectangle with one side a stone wall



Perimeter

$L + 2W = 400$

$2W = 400 - L$

$W = 200 - \frac{L}{2}$

Area $A = LW$

$A = L\left(200 - \frac{L}{2}\right)$

$A = 200L - \frac{L^2}{2}$

$\frac{dA}{dL} = 200 - L$

Max when $\frac{dA}{dL} = 0 \Rightarrow 200 - L = 0$

$200 = L$

$W = 200 - \frac{200}{2} = 100$

$\frac{d^2A}{dL^2} = -1 \quad \therefore$ a maximum at $L = 200$

Max Area $= L \times W$

$= 200 \times 100$

$= 20000\ m^2$

Exercise 12H

1e) $y = \frac{3x+8}{x^2} = \frac{3}{x} + \frac{8}{x^2} = 3x^{-1} + 8x^{-2}$

$\frac{dy}{dx} = -3x^{-2} - 16x^{-3} = -\frac{3}{x^2} - \frac{16}{x^3}$

$\frac{d^2y}{dx^2} = 6x^{-3} + 48x^{-4} = \frac{6}{x^3} + \frac{48}{x^4}$

---

4) $f(x) = px^3 - 3px^2 + x^2 - 4$

$f'(x) = 3px^2 - 6px + 2x$

$f''(x) = 6px - 6p + 2$

$f''(2) = 12p - 6p + 2 = -1$

$6p = -3$

$p = -\frac{1}{2}$

Homework Finish Exercise 12H