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5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy$$

- (a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

- (b) Find the gradient of the curve at each of these points.

(6)

a) When $x = -8$

$$(-8)^3 - 4y^2 = 12(-8)y$$

$$-512 - 4y^2 = -96y$$

$$4y^2 - 96y + 512 = 0$$

$$y^2 - 24y + 128 = 0$$

$$(y-8)(y-16) = 0$$

$$y = 8 \text{ or } y = 16$$

Points are $(-8, 8)$ and $(-8, 16)$

b)

$$x^3 - 4y^2 = 12xy$$

$$3x^2 - 8y \frac{dy}{dx} = 12x \frac{dy}{dx} + 12y$$

$$3x^2 - 12y = 12x \frac{dy}{dx} + 8y \frac{dy}{dx}$$

$$3x^2 - 12y = (12x + 8y) \frac{dy}{dx}$$



$$\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}$$

At $(-8, 8)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(-8)^2 - 12(8)}{12(-8) + 8(8)} \\ &= \frac{192 - 96}{-96 + 64} \\ &= -3\end{aligned}$$

At $(-8, 16)$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(-8)^2 - 12(16)}{12(-8) + 8(16)} \\ &= \frac{192 - 192}{-96 + 128} \\ &= 0\end{aligned}$$

At $(-8, 8)$ gradient = -3

At $(-8, 16)$ gradient = 0

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4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q .

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q .

(6)

(b) Find the coordinates of P and Q .

(3)

$$3x^2 - y^2 + xy = 4$$

a)

$$6x - 2y \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$6x + y = 2y \frac{dy}{dx} - x \frac{dy}{dx}$$

$$6x + y = (2y - x) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6x + y}{2y - x}$$

$$\text{If gradient} = \frac{8}{3}$$

$$\frac{8}{3} = \frac{6x + y}{2y - x}$$

$$8(2y - x) = 3(6x + y)$$

$$16y - 8x = 18x + 3y$$

$$13y = 26x$$

$$y = 2x$$

$$y - 2x = 0$$

\therefore at P, Q

$$\underline{y - 2x = 0}$$



$$b) \quad 3x^2 - y^2 + xy = 4$$

A t P, Q

$$y = 2x$$

$$3x^2 - (2x)^2 + x(2x) = 4$$

$$3x^2 - 4x^2 + 2x^2 = 4$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{when } x = 2, y = 4$$

$$x = -2, y = -4$$

so P, Q are the points $(2, 4)$ and $(-2, -4)$

1. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

a)

$$y^2 - 3y = x^3 + 8$$

$$2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx}(2y - 3) = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y - 3}$$

b)

$$\text{If } y = 3$$

$$3^2 - 3(3) = x^3 + 8$$

$$9 - 9 - 8 = x^3$$

$$\Rightarrow x = -2$$

Point on curve $(-2, 3)$

$$\frac{dy}{dx} = \frac{3(-2)^2}{2(3) - 3} = \frac{12}{3} = 4$$

Gradient = 4



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4. The curve C has the equation $ye^{-2x} = 2x + y^2$.

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (5)

The point P on C has coordinates $(0, 1)$.

- (b) Find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

a)

$$ye^{-2x} = 2x + y^2 \quad (4)$$

$$y(-2e^{-2x}) + e^{-2x}\frac{dy}{dx} = 2 + 2y\frac{dy}{dx}$$

$$e^{-2x}\frac{dy}{dx} - 2y\frac{dy}{dx} = 2 + 2ye^{-2x}$$

$$(e^{-2x} - 2y)\frac{dy}{dx} = 2 + 2ye^{-2x}$$

$$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$$

b) $P(0, 1)$ At P , $\frac{dy}{dx} = \frac{2+2}{1-2} = -4$

$$\text{Gradient of normal} = +\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{4}(x - 0)$$

$$4y - 4 = x$$

$$x - 4y + 4 = 0$$



3. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (3)

The point P lies on C where $x = \frac{\pi}{6}$.

- (b) Find the value of y at P . (3)

- (c) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c\pi = 0$, where a , b and c are integers. (3)

a)

$$\cos 2x + \cos 3y = 1$$

$$-2 \sin 2x - 3 \sin 3y \frac{dy}{dx} = 0$$

$$-2 \sin 2x = 3 \sin 3y \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{2 \sin 2x}{3 \sin 3y}$$

b) At P , $x = \frac{\pi}{6}$

$$\cos \frac{\pi}{3} + \cos 3y = 1$$

$$\frac{1}{2} + \cos 3y = 1$$

$$\cos 3y = \frac{1}{2}$$

$$3y = \frac{\pi}{3}$$

$$y = \frac{\pi}{9}$$



Question 3 continued

c) $P\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

$$\text{At } P, \quad \frac{dy}{dx} = -\frac{2 \sin \frac{\pi}{3}}{3 \sin \frac{\pi}{2}} = -\frac{2}{3}$$

Tangent at P

$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{3} = -\frac{2}{3}(x - \frac{\pi}{6})$$

$$3y - \frac{\pi}{3} = -2(x - \frac{\pi}{6})$$

$$3y - \frac{\pi}{3} = -2x + \frac{2\pi}{3}$$

$$2x + 3y - \frac{2\pi}{3} = 0$$

$$6x + 9y - 2\pi = 0$$



N 3 5 3 8 2 A 0 9 2 8

3. A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

Aside

$$\text{If } y = 2^x$$

$$\ln y = \ln 2^x$$

$$\ln y = x \ln 2$$

$$\frac{1}{y} \frac{dy}{dx} = \ln 2$$

$$\frac{dy}{dx} = y \ln 2$$

$$\frac{dy}{dx} = 2^x \ln 2$$

$$2^x + y^2 = 2xy \quad (7)$$

$$2^x \ln 2 + 2y \frac{dy}{dx} = 2x \frac{dy}{dx} + 2y$$

$$2^x \ln 2 - 2y = 2x \frac{dy}{dx} - 2y \frac{dy}{dx}$$

$$2^x \ln 2 - 2y = (2x - 2y) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2^x \ln 2 - 2y}{2x - 2y}$$

At $(3, 2)$

$$\frac{dy}{dx} = \frac{2^3 \ln 2 - 2(2)}{2(3) - 2(2)}$$

$$\frac{dy}{dx} = \frac{8 \ln 2 - 4}{6 - 4}$$

$$\frac{dy}{dx} = 4 \ln 2 - 2$$

