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7. A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor's records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

(2)

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients.

(2)

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

(c) Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.

(6)

(d) From a sample of size 20, find the greatest number of patients who need to recover for the test in part (c) to be significant at the 1% level.

(4)

a) $X \sim B(10, 0.75)$

b) $P(X=6) = {}^{10}C_6 \times 0.75^6 \times 0.25^4 = 0.1460$

or use Binomial PD on calculator to calculate directly

c) $H_0 : p = 0.75$

$H_1 : p < 0.75$ where p is probability a randomly chosen patient recovers when treated with new drug

$X \sim B(20, 0.75)$

$P(X \leq 13) = 0.2142 > 5\%$

using Binomial CD on calculator

Leave blank

Question 7 continued

Accept Ho. There is insufficient evidence to suggest that less than 75% recover. So accept that 75% recover.

d) $P(X \leq 9) = 0.0039 \quad < 1\%$

$P(X \leq 10) = 0.0138 \quad > 1\%$

using list on Binomial CD on calculator

9 is the greatest number of patients that can recover if result is to be significant.

Leave blank

7. It is known from past records that 1 in 5 bowls produced in a pottery have minor defects. To monitor production a random sample of 25 bowls was taken and the number of such bowls with defects was recorded.

(a) Using a 5% level of significance, find critical regions for a two-tailed test of the hypothesis that 1 in 5 bowls have defects. The probability of rejecting, in either tail, should be as close to 2.5% as possible.

(6)

(b) State the actual significance level of the above test.

(1)

At a later date, a random sample of 20 bowls was taken and 2 of them were found to have defects.

(c) Test, at the 10% level of significance, whether or not there is evidence that the proportion of bowls with defects has decreased. State your hypotheses clearly.

(7)

a) $X \sim B(25, 0.2)$

$P(X=0) = 0.0038 < 2\frac{1}{2}\%$

$P(X \leq 1) = 0.0273 > 2\frac{1}{2}\%$ closest to $2\frac{1}{2}\%$

Bottom end critical region $(X \leq 1)$

closest to $2\frac{1}{2}\%$

$P(X \leq 9) = 0.9826 \Rightarrow P(X \geq 10) = 0.0174 < 2\frac{1}{2}\%$

$P(X \leq 8) = 0.9532 \Rightarrow P(X \geq 9) = 0.0468 > 2\frac{1}{2}\%$

Top end critical region $(X \geq 10)$

ie it is 10 that takes you into top $2\frac{1}{2}\%$

Critical Region $(X \leq 1) \cup (X \geq 10)$

b) Actual significance level $= 0.0174 + 0.0273$

$= 0.0447$

$= 4.47\%$

N 2 2 3 3 8 A 0 2 2 2 4

Leave blank

Question 7 continued

c) $X \sim B(20, 0.2)$

$H_0 : p = 0.2$

$H_1 : p < 0.2$ where p = probability a randomly chosen bowl has a defect

$P(X \leq 2) = 0.206 > 10\%$

Accept H_0

There is not sufficient evidence to suggest the proportion of bowls with defects has decreased.

Leave blank

6. Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

(7)

$X \sim B(5, 0.2)$

$H_0 : p = 0.2$

$H_1 : p > 0.2$ where p is prob taxi is late on randomly chosen day

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.9421$

$= 0.0579 > 5\%$ Accept H_0

There is not sufficient evidence to suggest that the taxi is late more than once a week.

N 2 6 1 1 9 A 0 1 4 2 4

Leave blank

5. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head. **(2)**

Find the probability that Sue records

(b) exactly 8 heads, **(2)**

(c) at least 4 heads. **(2)**

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly. **(6)**

a) $X \sim B(15, 0.5)$

b) $P(X=8) = {}^{15}C_8 \times 0.5^8 \times 0.5^7 = 0.1964$

c) $P(X \geqslant 4) = 1 - P(X \leq 3) = 1 - 0.0176 = 0.9824$

d) $X \sim B(15, 0.5)$

$H_0 : p = 0.5$

$H_1 : p > 0.5$ where p is prob of obtaining a head with this coin on a randomly chosen spin

$P(X \geqslant 13) = 1 - P(X \leq 12)$

$= 1 - 0.9963$

$= 0.0037 < 1\%$

Reject H_0 and accept H_1

There is sufficient evidence to suggest new coin is biased in favour of heads.

H 3 2 5 8 1 A 0 1 4 2 8

Leave blank

4. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05.

(5)

(b) Write down the actual significance level of a test based on your critical region from part (a).

(1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a).

(2)

a) $X \sim B(20, 0.3)$

$H_0 : p = 0.3$

$H_1 : p <> 0.3$

where p is prob a random customer buys 1 tin when buying beans

Bottom $P(X \leq 2) = 0.0354 < 5\%$

$P(X \leq 3) = 0.107$

Top $P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.952 = 0.048 < 5\%$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.8866 = 0.1134$

Critical Region $(X \leq 2) \cup (X \geq 10)$

Prob rejection at bottom $= 0.0354$

Prob rejection at top $= 0.048$

b) Actual significance level $0.0354 + 0.048 = 8.34\%$

c) 11 is in the critical region, so reject H_0, accept H_1

There is sufficient evidence to suggest that the proportion of customers buying beans who buy 1 tin has changed from 30%