1 GCSE VECTORS

26

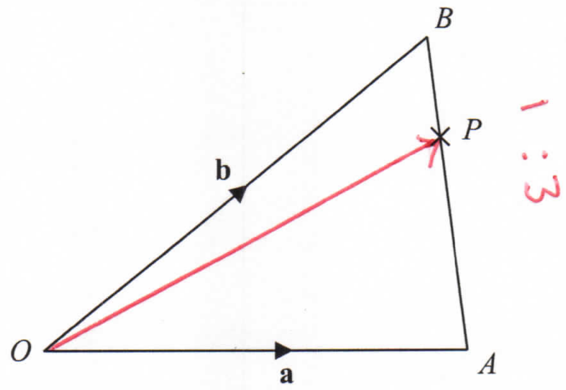

Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{a} + \underline{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} =$ $\underline{b} - \underline{a}$

(1)

P is the point on *AB* such that $AP : PB = 3 : 1$

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB}$

$= \underline{a} + \frac{3}{4}(\underline{b} - \underline{a})$

$= \underline{a} + \frac{3}{4}\underline{b} - \frac{3}{4}\underline{a}$

$= \frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$

$\overrightarrow{OP} =$ $\frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$

(3)

(Total for Question 26 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

GCSE VECTORS

28

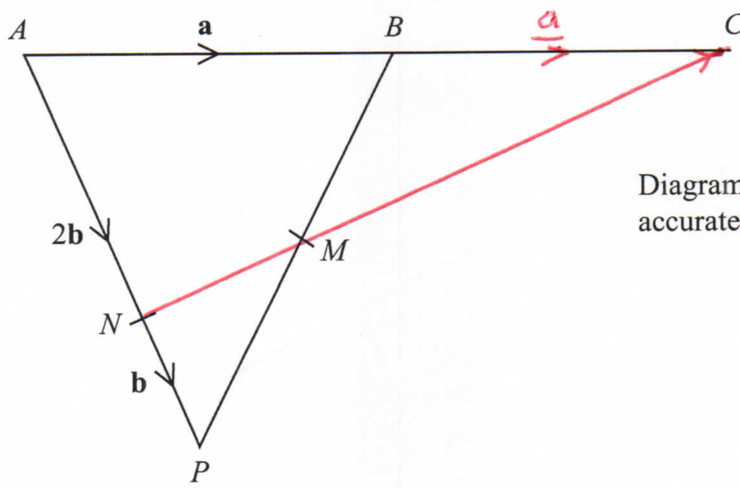

Diagram **NOT** accurately drawn

APB is a triangle.
N is a point on *AP*.

$\overrightarrow{AB} = \mathbf{a}$ $\qquad \overrightarrow{AN} = 2\mathbf{b}$ $\qquad \overrightarrow{NP} = \mathbf{b}$

(a) Find the vector $\overrightarrow{PB}$, in terms of **a** and **b**.

$\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$

$= -3\underline{b} + \underline{a}$

$\overrightarrow{PB} = \underline{a} - 3\underline{b}$

(1)

B is the midpoint of *AC*.
M is the midpoint of *PB*.

*(b) Show that *NMC* is a straight line.

$\overrightarrow{NM} = \overrightarrow{NP} + \overrightarrow{PM}$

$= \underline{b} + \frac{1}{2}(\underline{a} - 3\underline{b})$

$= \underline{b} + \frac{1}{2}\underline{a} - \frac{3}{2}\underline{b}$

$= \frac{1}{2}\underline{a} - \frac{1}{2}\underline{b} \qquad = \frac{1}{2}(\underline{a} - \underline{b})$

$\overrightarrow{NC} = \overrightarrow{NA} + \overrightarrow{AC}$

$= -2\underline{b} + 2\underline{a} = 2(-\underline{b} + \underline{a}) = 2(\underline{a} - \underline{b})$

$\therefore$ $\overrightarrow{NM}$ and $\overrightarrow{NC}$ are in same direction

So NMC is a straight line.

(4)

(Total for Question 28 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

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GCSE VECTORS

26

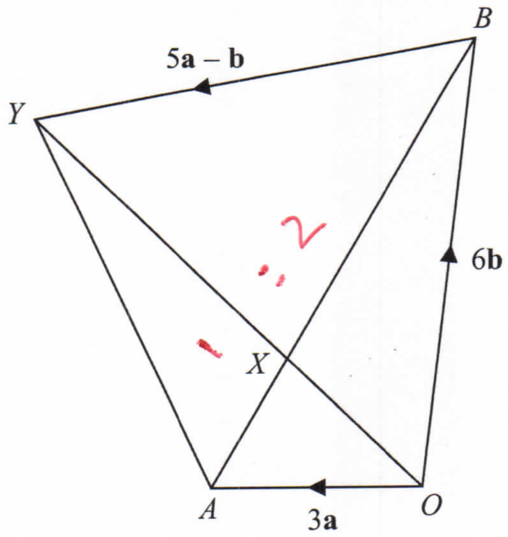

Diagram **NOT** accurately drawn

OAYB is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$

$\overrightarrow{OB} = 6\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3\underline{a} + 6\underline{b}$

$\overrightarrow{AB} =$ $6\underline{b} - 3\underline{a}$

(1)

X is the point on *AB* such that $AX : XB = 1 : 2$

and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

$\overrightarrow{OY} = \overrightarrow{OB} + \overrightarrow{BY}$

$= 6\underline{b} + 5\underline{a} - \underline{b}$

$= 5\underline{b} + 5\underline{a} = 5(\underline{a} + \underline{b})$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$

$= 3\underline{a} + \frac{1}{3}(6\underline{b} - 3\underline{a})$

$= 3\underline{a} + 2\underline{b} - \underline{a}$

$= 2\underline{a} + 2\underline{b} = 2(\underline{a} + \underline{b})$

$\therefore \overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

(4)

(Total for Question 26 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

P 4 2 0 5 7 A 0 2 7 2 8

GCSE VECTORS

27

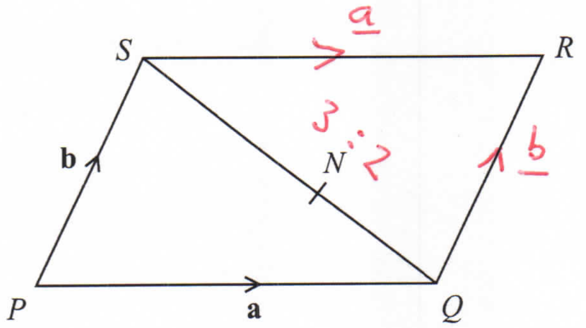

Diagram **NOT** accurately drawn

PQRS is a parallelogram.
N is the point on *SQ* such that $SN : NQ = 3 : 2$
$\overrightarrow{PQ} = \mathbf{a}$
$\overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

$\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ}$
$= -\underline{b} + \underline{a}$

$\overrightarrow{SQ} =$ $\underline{a} - \underline{b}$
(1)

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

$\overrightarrow{NR} = \overrightarrow{NQ} + \overrightarrow{QR}$
$= \frac{2}{5}\overrightarrow{SQ} + \overrightarrow{QR}$
$= \frac{2}{5}(\underline{a} - \underline{b}) + \underline{b}$
$= \frac{2}{5}\underline{a} - \frac{2}{5}\underline{b} + \underline{b}$
$= \frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$

$\overrightarrow{NR} =$ $\frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$
(3)

(Total for Question 27 is 4 marks)

P 4 3 5 9 8 A 0 2 5 2 8

Turn over ▶

GCSE VECTORS

24 *OACB* is a parallelogram.

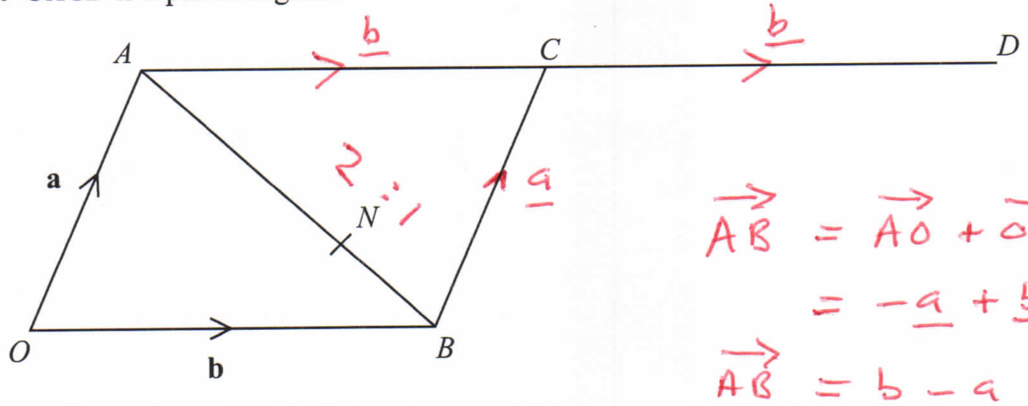

Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$
D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$
The point N divides AB in the ratio $2:1$

(a) Write an expression for $\overrightarrow{ON}$ in terms of **a** and **b**.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -\underline{a} + \underline{b}$$
$$\overrightarrow{AB} = \underline{b} - \underline{a}$$
$$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$$
$$= \overrightarrow{OA} + \tfrac{2}{3}\overrightarrow{AB}$$
$$= \underline{a} + \tfrac{2}{3}(\underline{b} - \underline{a})$$
$$= \underline{a} + \tfrac{2}{3}\underline{b} - \tfrac{2}{3}\underline{a}$$
$$\overrightarrow{ON} = \tfrac{1}{3}\underline{a} + \tfrac{2}{3}\underline{b}$$
$$= \tfrac{1}{3}(\underline{a} + 2\underline{b})$$

....

(3)

*(b) Prove that *OND* is a straight line.

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= \underline{a} + 2\underline{b} \qquad = 3\overrightarrow{ON}$$

$\therefore$ $\overrightarrow{OD}$ is in same direction as $\overrightarrow{ON}$

so OND is a straight line.

(3)

(Total for Question 24 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

P 4 3 3 8 3 A 0 2 7 2 8

GCSE VECTORS

*24

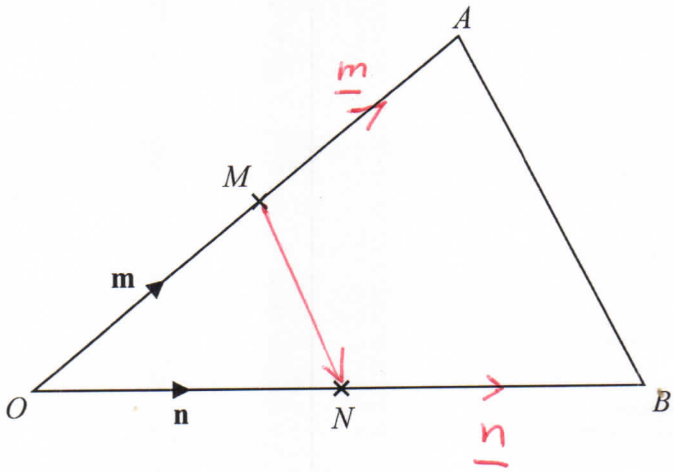

Diagram **NOT** accurately drawn

OAB is a triangle.
M is the midpoint of *OA*.
N is the midpoint of *OB*.

$\overrightarrow{OM} = \mathbf{m}$

$\overrightarrow{ON} = \mathbf{n}$

Show that *AB* is parallel to *MN*.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -2\underline{m} + 2\underline{n}$

$= 2(\underline{n} - \underline{m})$

$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$

$= -\underline{m} + \underline{n} = \underline{n} - \underline{m}$

$\overrightarrow{AB} = 2\,\overrightarrow{MN}$

$\therefore$ $\overrightarrow{AB}$ and $\overrightarrow{MN}$ are parallel.

(Total for Question 24 is 3 marks)

GCSE VECTORS

27

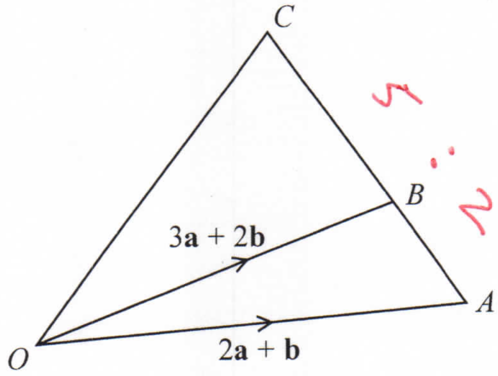

Diagram **NOT** accurately drawn

ABC is a straight line.

$AB : BC = 2 : 5$

$\overrightarrow{OA} = 2\mathbf{a} + \mathbf{b}$

$\overrightarrow{OB} = 3\mathbf{a} + 2\mathbf{b}$

Express $\overrightarrow{OC}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -2\underline{a} - \underline{b} + 3\underline{a} + 2\underline{b}$

$\overrightarrow{AB} = \underline{a} + \underline{b}$

$\overrightarrow{AC} = \frac{7}{2}(\underline{a} + \underline{b})$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= 2\underline{a} + \underline{b} + \frac{7}{2}\underline{a} + \frac{7}{2}\underline{b}$

$= \frac{11}{2}\underline{a} + \frac{9}{2}\underline{b}$

....

(Total for Question 27 is 4 marks)

P 4 5 8 7 2 A 0 2 4 2 8

GCSE VECTORS

*20

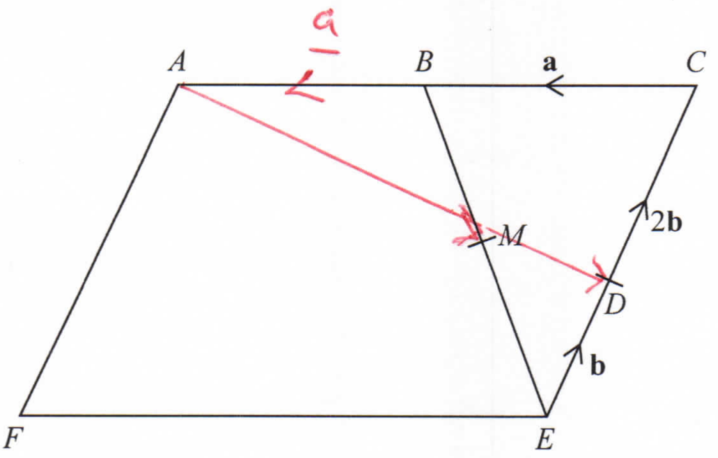

Diagram **NOT** accurately drawn

ACEF is a parallelogram.
B is the midpoint of *AC*.
M is the midpoint of *BE*.

$\overrightarrow{CB} = \mathbf{a}$

$\overrightarrow{ED} = \mathbf{b}$

$\overrightarrow{DC} = 2\mathbf{b}$

Show that *AMD* is a straight line.

$$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$$
$$= -2\underline{a} - 2\underline{b}$$
$$= -2(\underline{a} + \underline{b})$$

$$\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$$
$$= -\underline{a} - 3\underline{b}$$

$$\overrightarrow{BM} = -\tfrac{1}{2}\underline{a} - \tfrac{3}{2}\underline{b}$$

$$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$$
$$= -\underline{a} - \tfrac{1}{2}\underline{a} - \tfrac{3}{2}\underline{b}$$

$$\overrightarrow{AM} = -\tfrac{3}{2}\underline{a} - \tfrac{3}{2}\underline{b} = -\tfrac{3}{2}(\underline{a} + \underline{b})$$

$\overrightarrow{AD}$ is therefore a multiple of $\overrightarrow{AM}$

$\therefore$ AMD is a straight line.

(Total for Question 20 is 5 marks)

GCSE VECTORS

23

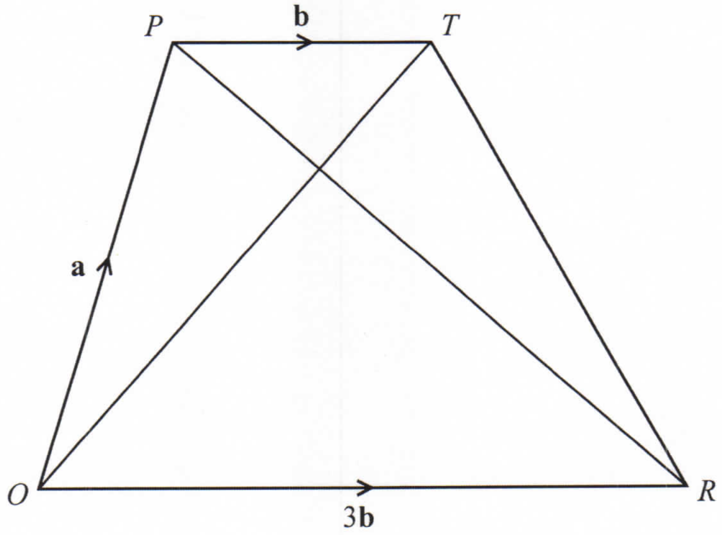

Diagram **NOT** accurately drawn

OPTR is a trapezium.

$\overrightarrow{OP} = \mathbf{a}$
$\overrightarrow{PT} = \mathbf{b}$
$\overrightarrow{OR} = 3\mathbf{b}$

(a) (i) Find $\overrightarrow{OT}$ in terms of **a** and **b**

$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT}$

$= \underline{a} + \underline{b}$

....

(ii) Find $\overrightarrow{PR}$ in terms of **a** and **b**
Give your answer in its simplest form.

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}$

$= -\underline{a} + 3\underline{b}$

$= 3\underline{b} - \underline{a}$

....

(2)

DO NOT WRITE IN THIS AREA

P 4 9 3 0 2 A 0 2 4 2 8

GCSE VECTORS

23

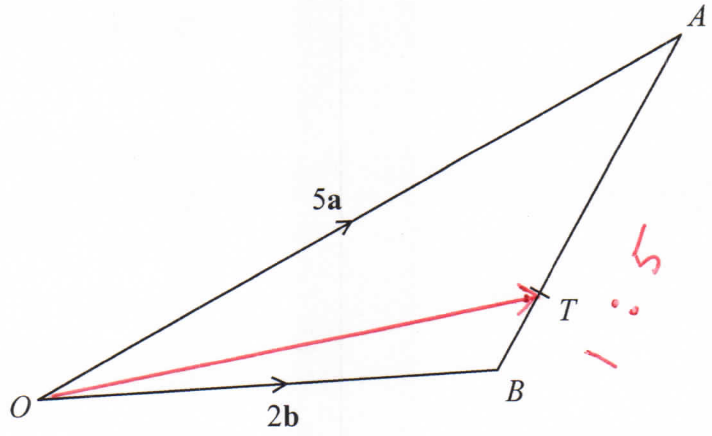

Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = 5\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

T is the point on *AB* such that *AT* : *TB* = 5 : 1

Show that *OT* is parallel to the vector $\mathbf{a} + 2\mathbf{b}$

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$

$= -2\underline{b} + 5\underline{a}$

$\overrightarrow{BT} = \frac{1}{2}\overrightarrow{BA}$

$\overrightarrow{BT} = -\frac{2}{6}\underline{b} + \frac{5}{6}\underline{a}$

$\overrightarrow{OT} = \overrightarrow{OB} + \overrightarrow{BT}$

$= 2\underline{b} - \frac{2}{6}\underline{b} + \frac{5}{6}\underline{a}$

$= \frac{10}{6}\underline{b} + \frac{5}{6}\underline{a} \qquad = \frac{5}{6}(\underline{a} + 2\underline{b})$

$\therefore \overrightarrow{OT}$ is parallel to $\underline{a} + 2\underline{b}$

(Total for Question 23 is 4 marks)

P 4 8 1 9 6 A 0 2 6 2 8