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4.

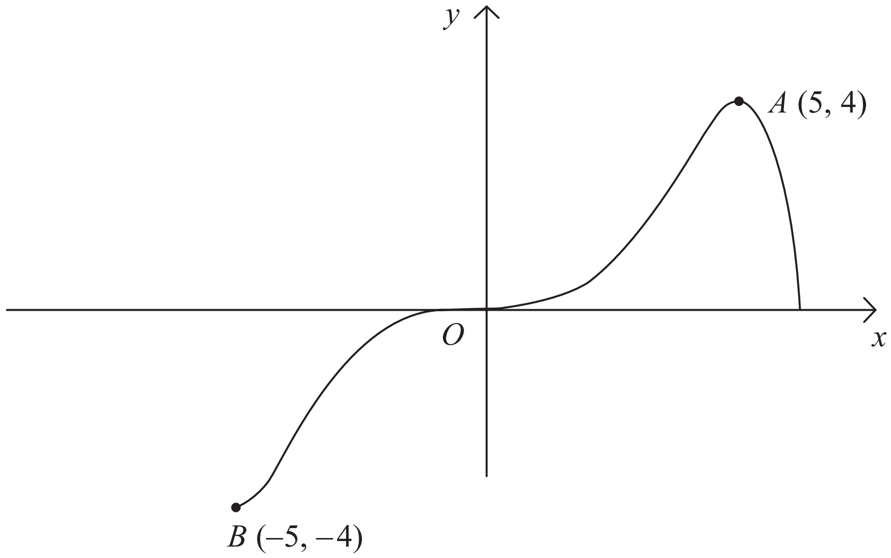


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}(|x|)$,

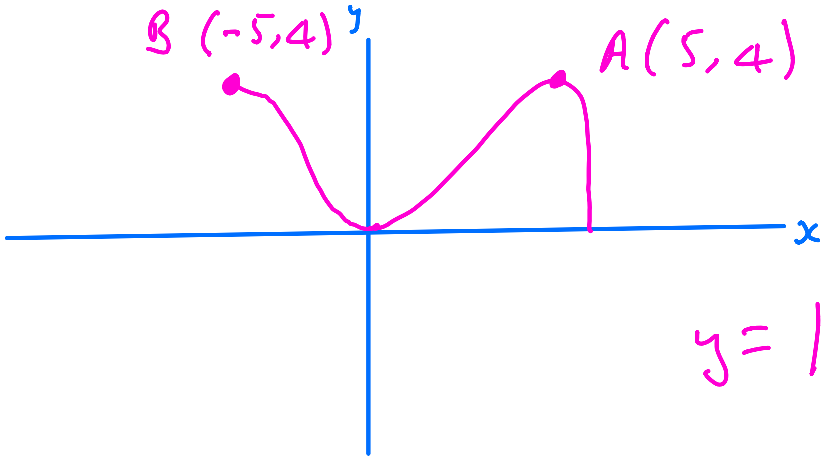
(3)

(c) $y = 2\mathrm{f}(x+1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

a)



$y = |f(x)|$

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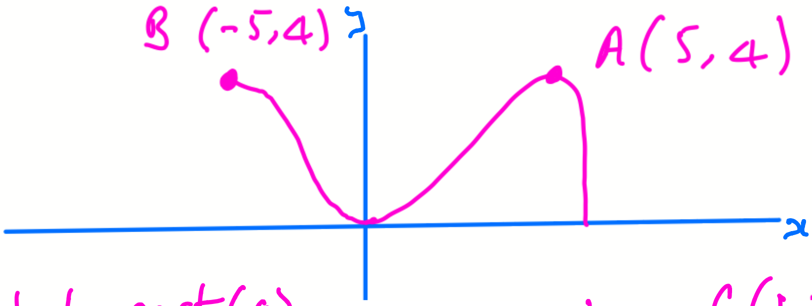
Question 4 continued

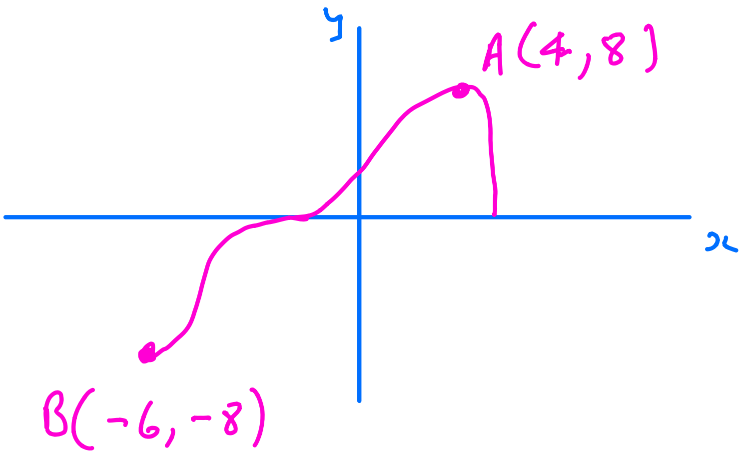
b)



Identical to part (a)

$y = f(|x|)$

c)



H 2 6 3 1 5 R B 0 9 2 4

Turn over

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8. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$\mathrm{g} : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$\mathrm{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $\mathrm{gf}(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

(5)

a) Let $y = 1 - 2x^3$

swap variables $x = 1 - 2y^3$

$$2y^3 = 1 - x$$

$$y^3 = \frac{1-x}{2}$$

$$y = \sqrt[3]{\frac{1-x}{2}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{1-x}{2}}$$

b) $gf(x) = g(1-2x^3)$

$$= \frac{3}{(1-2x^3)} - 4$$

Leave blank

Question 8 continued

$$gf(x) = \frac{3-4(1-2x^3)}{(1-2x^3)}$$

$$= \frac{3-4+8x^3}{(1-2x^3)}$$

$$= \frac{8x^3-1}{(1-2x^3)}$$

c) $gf(x) = 0 \Rightarrow \frac{8x^3-1}{(1-2x^3)} = 0$

$$\Rightarrow 8x^3 - 1 = 0$$

$$\Rightarrow x^3 = \frac{1}{8}$$

$$\Rightarrow x = \frac{1}{2}$$

d) $y = \frac{8x^3-1}{(1-2x^3)}$ $\quad \frac{dy}{dx} = \frac{(1-2x^3)(24x^2)-(8x^3-1)(-6x^2)}{(1-2x^3)^2}$

st pt when $\frac{dy}{dx} = 0$ $\Rightarrow 24x^2 - 48x^5 + 48x^5 - 6x^2 = 0$

$$\Rightarrow 18x^2 = 0$$

$$\Rightarrow x = 0$$

$$\Rightarrow y = \frac{0-1}{1-0} = -1$$

st pt at $(0, -1)$

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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3.

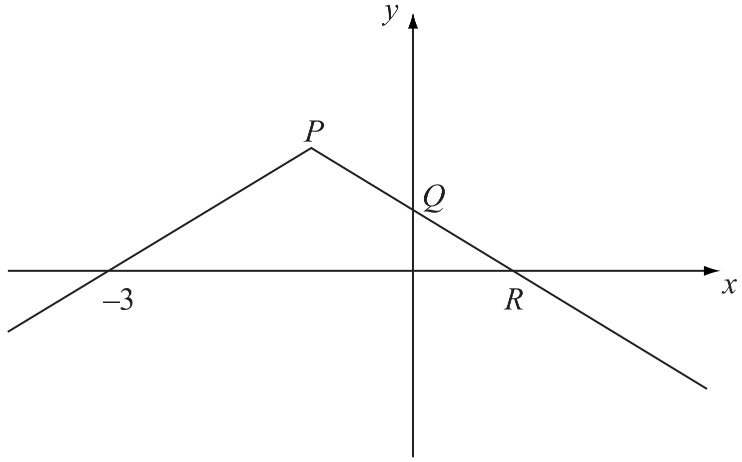


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.
Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$, **(2)**

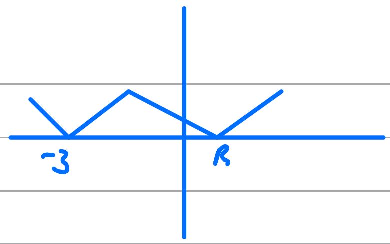
(b) $y = \mathrm{f}(-x)$. **(2)**

Given that $\mathrm{f}(x) = 2 - |x+1|$,

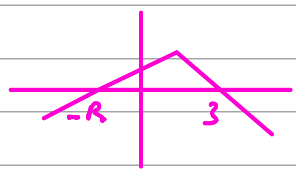
(c) find the coordinates of the points P, Q and R, **(3)**

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$. **(5)**

a) $y = |f(x)|$



b) $y = f(-x)$



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Question 3 continued

c) $f(x) = 2 - |x+1|$

At R $\quad 2 - |x+1| = 0 \quad \Rightarrow x = 1$ or -3

$R(1, 0)$

At Q $\quad x = 0 \quad f(x) = 2 - |0+1| = 1$

$Q(0, 1)$

At P $\quad |x+1| = 0$

$x = -1 \quad f(-1) = 2 - |-1+1| = 2$

$P(-1, 2)$

d) $f(x) = \frac{1}{2}x \qquad 2 - |x+1| = \frac{1}{2}x$

$2 - \frac{1}{2}x = |x+1|$

$x + 1 = 2 - \frac{1}{2}x$ or $-x - 1 = 2 - \frac{1}{2}x$

$\frac{3}{2}x = 1 \qquad -3 = \frac{1}{2}x$

$x = \frac{2}{3}$ or $x = -6$

Leave blank

4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \dfrac{1}{x+1}, \quad x > 3.$ **(4)**

(b) Find the range of f. **(2)**

(c) Find $f^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \dfrac{1}{8}$. **(3)**

a) $f(x) = \dfrac{2(x-1)}{x^2-2x-3} - \dfrac{1}{x-3}$

$= \dfrac{2(x-1)}{(x+1)(x-3)} - \dfrac{1}{(x-3)}$

$= \dfrac{2x-2-(x+1)}{(x+1)(x-3)}$

$= \dfrac{2x-2-x-1}{(x+1)(x-3)}$

$= \dfrac{(x-3)}{(x+1)(x-3)}$

$= \dfrac{1}{x+1}$

b) Range of f $\quad 0 < f(x) < \frac{1}{4}$

Set notation $\quad$ Range of $f(x) = \left(0, \frac{1}{4}\right)$

Leave blank

Question 4 continued

c) Let $y = \frac{1}{x+1}$

Swap variables $x = \frac{1}{y+1}$

$y+1 = \frac{1}{x}$

$y = \frac{1}{x} - 1$

$y = \frac{1-x}{x}$

$f^{-1}(x) = \frac{1-x}{x}$

domain of $f^{-1}(x)$ $0 < x < \frac{1}{4}$ or $\left(0, \frac{1}{4}\right)$

d) $f(x) = \frac{1}{x+1}$ $g(x) = 2x^2 - 3$

$fg(x) = f(2x^2-3) = \frac{1}{2x^2-3+1}$

$= \frac{1}{2x^2-2}$

Solve $fg(x) = \frac{1}{8}$

$\frac{1}{8} = \frac{1}{2x^2-2}$

$\Rightarrow 2x^2 - 2 = 8$

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

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3.

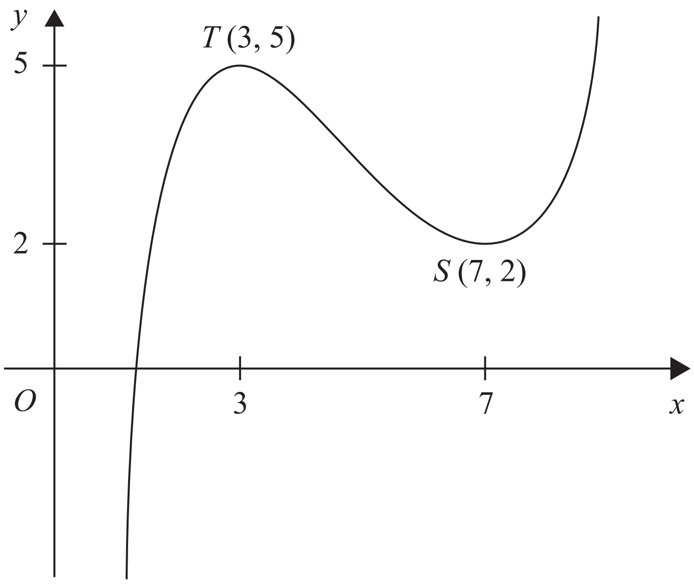


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $\quad 1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x) - 4$,

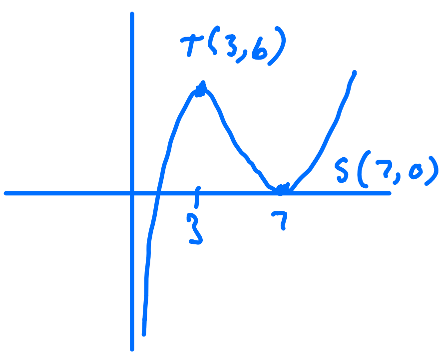
(3)

(b) $y = |\mathrm{f}(x)|$.

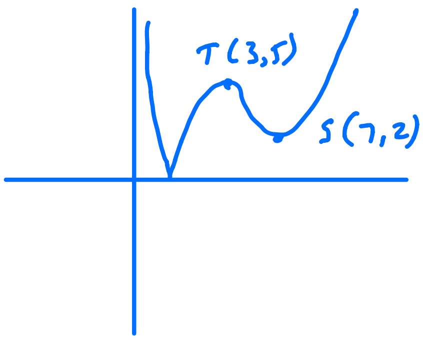
(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

$y = 2f(x) - 4$



$y = |f(x)|$



H 3 1 1 2 3 A 0 8 2 8

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5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g. **(1)**

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg. **(1)**

(d) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$. **(6)**

a) Range of g $\quad g(x) \geq 1$

or $\{x : x \geq 1, x \in \mathbb{R}\}$

b) $f(x) = 3x + \ln x \qquad g(x) = e^{x^2}$

$fg(x) = f(e^{x^2})$

$fg(x) = 3e^{x^2} + \ln e^{x^2}$

$= 3e^{x^2} + x^2 \ln e$

$= 3e^{x^2} + x^2$

c) Range $\quad fg(x) \geq 3$

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Question 5 continued

d) $\frac{d}{dx} fg(x) = 6xe^{x^2} + 2x$

Solve $6xe^{x^2} + 2x = x(xe^{x^2} + 2)$

$6xe^{x^2} + 2x = x^2e^{x^2} + 2x$

$6xe^{x^2} = x^2e^{x^2}$

$(6x - x^2)e^{x^2} = 0$

$x(6-x)e^{x^2} = 0$

$\Rightarrow x = 0 \text{ or } x = 6$

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5.

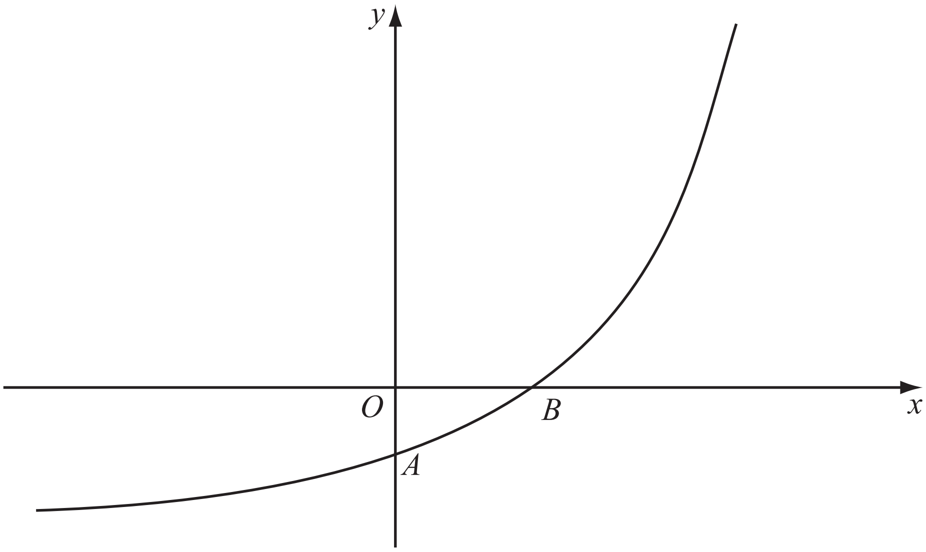

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$,

(3)

(b) $y = f^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f ,

(1)

(d) find $f^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

(1)

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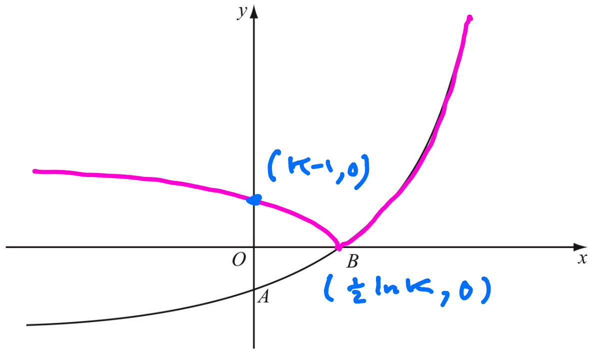
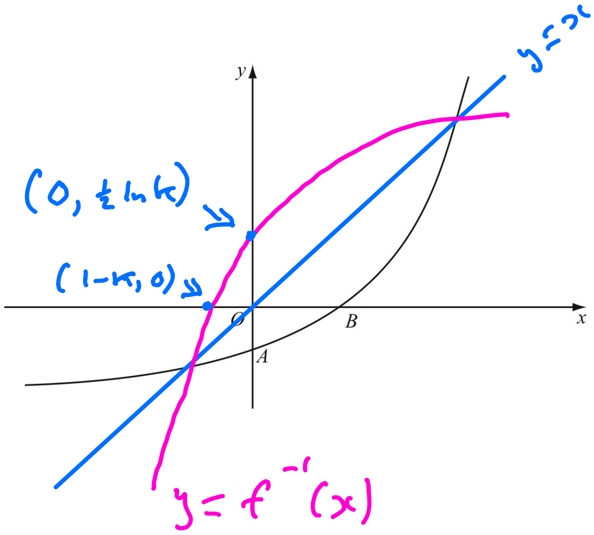
Question 5 continued

a)



$y = |f(x)|$

b)



reflection of $y = f(x)$ in line $y = x$

c) Range $f(x) > -k$ or $(-k, \infty)$

d)

$y = e^{2x} - k$

swap $x = e^{2y} - k$

$x + k = e^{2y}$

$\ln(x+k) = 2y$

$y = \frac{1}{2}\ln(x+k)$

$f^{-1}(x) = \frac{1}{2}\ln(x+k)$

e) Domain of $f^{-1}(x)$

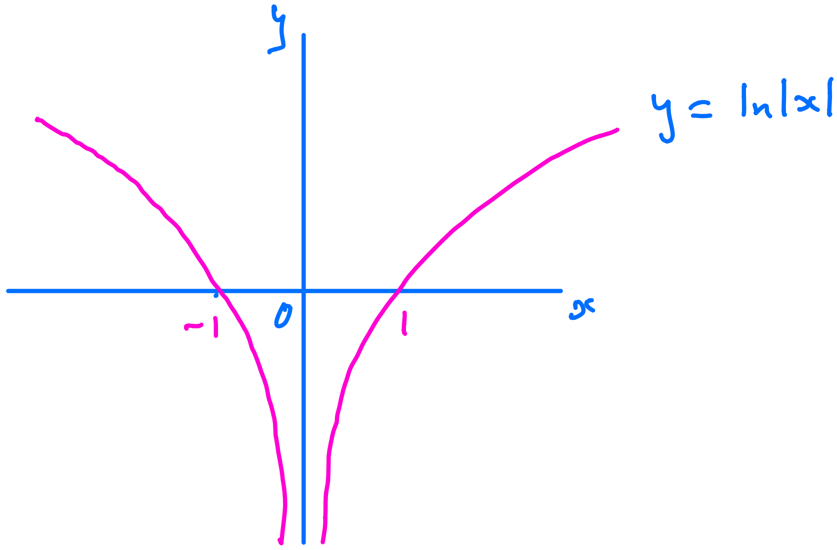
$x > -k$

or $(-k, \infty)$

Leave blank

5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)



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6.

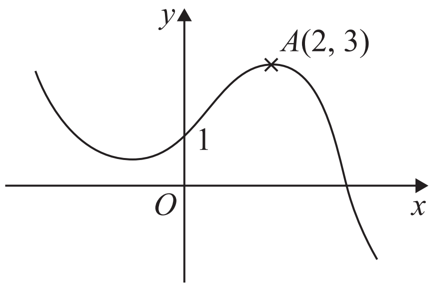


Figure 1

Figure 1 shows a sketch of the graph of $y = \mathrm{f}(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = \mathrm{f}(-x) + 1$,

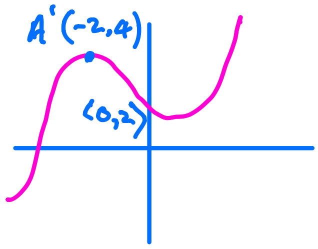
(ii) $y = \mathrm{f}(x + 2) + 3$,

(iii) $y = 2\mathrm{f}(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

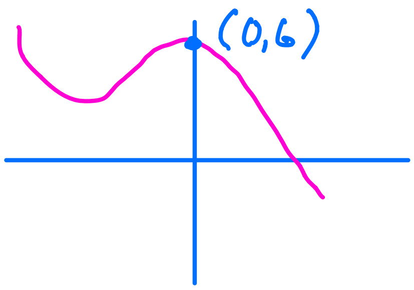
(9)

i)



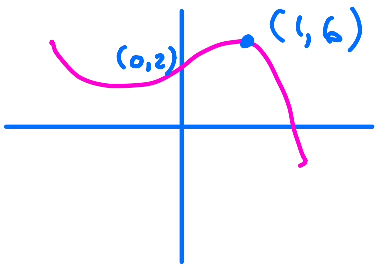
$y = f(-x) + 1$

ii)



$y = f(x+2) + 3$

iii)



$y = 2f(2x)$

Leave blank

Question 6 continued

N 3 5 3 8 1 A 0 1 5 2 8

Turn over

Leave blank

9. (i) Find the exact solutions to the equations

(a) $\ln(3x-7)=5$ **(3)**

(b) $3^x e^{7x+2}=15$ **(5)**

(ii) The functions f and g are defined by

$$f(x)=e^{2x}+3, \qquad x\in\mathbb{R}$$
$$g(x)=\ln(x-1), \qquad x\in\mathbb{R},\ x>1$$

(a) Find f^{-1} and state its domain. **(4)**

(b) Find fg and state its range. **(3)**

i) a) $\ln(3x-7)=5$

$3x-7=e^5$

$3x=e^5+7$ $\qquad x=\dfrac{e^5+7}{3}$

b) $3^x e^{7x+2}=15$

$\ln\left(3^x e^{7x+2}\right)=\ln 15$

$\ln 3^x+\ln e^{7x+2}=\ln 15$

$x\ln 3+7x+2=\ln 15$

$x(7+\ln 3)=\ln 15-2$

$x=\dfrac{\ln 15-2}{7+\ln 3}$

ii a) $f(x)=e^{2x}+3$

$y=e^{2x}+3$

swap $x=e^{2y}+3$

$x-3=e^{2y}$

Leave blank

Question 9 continued

$\ln(x-3) = 2y$

$f^{-1}(x) = \frac{1}{2}\ln(x-3)$

domain of $f^{-1}(x)$ $\quad x > 3$

b)

$f(x) = e^{2x} + 3 \qquad g(x) = \ln(x-1)$

$fg(x) = f(\ln(x-1))$

$= e^{2\ln(x-1)} + 3$

$= e^{\ln(x-1)^2} + 3$

$fg(x) = (x-1)^2 + 3$

Range $\quad fg(x) > 3$

Q9

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.
(2)

(b) Solve $f(x) = 15 + x$.
(3)

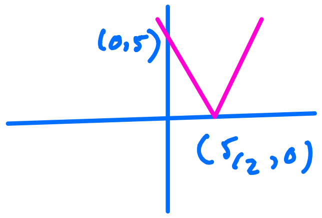
The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find fg(2).
(2)

(d) Find the range of g.
(3)

a)



b) $|2x-5| = 15 + x$

Either $2x - 5 = 15 + x$ or $-2x + 5 = 15 + x$

$\underline{x = 20}$

$-10 = 3x$

$\underline{x = -\frac{10}{3}}$

c) $f(x) = |2x - 5|$ $\quad g(x) = x^2 - 4x + 1$

$fg(x) = f(x^2 - 4x + 1)$

$= |2x^2 - 8x + 2 - 5|$

$= |2x^2 - 8x - 3|$

Leave blank

Question 4 continued

$$fg(2) = |2(2)^2 - 8(2) - 3|$$

$$= |8 - 16 - 3|$$

$$= |-11|$$

$$= 11$$

d)

$$g(x) = x^2 - 4x + 1$$

$$= (x-2)^2 + 1 - 4$$

$$= (x-2)^2 - 3$$

$$\therefore \quad g(x) \geq -3$$

but $0 \leq x \leq 5$

$$g(5) = (5-2)^2 - 3 = 6$$

$$g(2) = 0^2 - 3 = -3$$

Range $-3 \leq g(x) \leq 6$

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6.

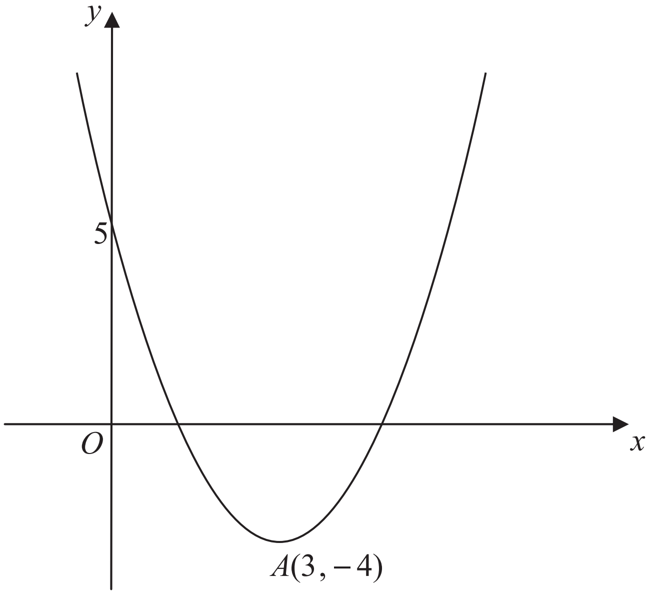

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |\mathrm{f}(x)|$, A′(3, 4)

(ii) $y = 2\mathrm{f}(\frac{1}{2}x)$. A′(6, −8)

(4)

(b) Sketch the curve with equation

$$y = \mathrm{f}(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = \mathrm{f}(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $\mathrm{f}(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

H 3 5 3 8 5 A 0 1 8 2 8

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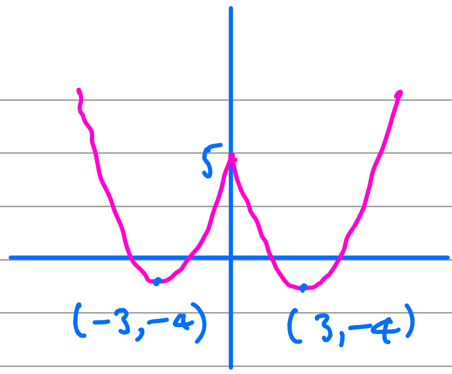
Question 6 continued

b)



c) Translation by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$$f(x) = (x-3)^2 - 4$$

d) No inverse since many to one

Must be one to one to have an inverse