EDEXCEL FP1 COMPLEX NUMBERS 2010 − 2016

JAN 2010

1) $z_1 = 2 + 8i \qquad z_2 = 1 - i$

a) $\dfrac{z_1}{z_2} = \dfrac{2+8i}{1-i}$

$= \dfrac{2+8i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{2 + 8i + 2i + 8i^2}{1^2 + 1^2}$

$= \dfrac{-6 + 10i}{2}$

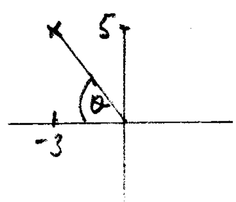
$= -3 + 5i$

b) $\left|\dfrac{z_1}{z_2}\right| = \sqrt{(-3)^2 + 5^2}$

$= \sqrt{34}$

or 5.83 to 3 s.f.

c)



$\arg\left(\dfrac{z_1}{z_2}\right) = \pi - \theta$

$= \pi - \tan^{-1}\left(\dfrac{5}{3}\right)$

$= 2.11$ to 2 d.p.

JAN 2010

6) 2 and $5 + 2i$ are roots

a) Other root is $5 - 2i$

b) $x^3 - 12x^2 + cx + d = 0$

$\Sigma \alpha\beta = c$

$\Rightarrow 2(5+2i) + 2(5-2i) + (5+2i)(5-2i) = c$

$\Rightarrow 10 + \cancel{4i} + 10 - \cancel{4i} + 5^2 + 2^2 = c$

$\Rightarrow 49 = c$

$c = 49$

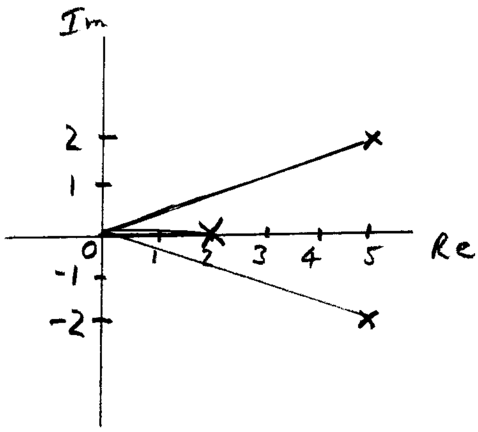
$\alpha\beta\gamma = -d$

$\Rightarrow 2(5+2i)(5-2i) = -d$

$58 = -d$

$d = -58$

c)



EDEXCEL FP1 COMPLEX NUMBERS 2010 – 2016

JUN 2010

1) $z = 2 - 3i$

a) $z^2 = (2-3i)(2-3i)$

$= 4 - 6i - 6i + 3i^2$

$= 4 - 12i - 9$

$= -5 - 12i$

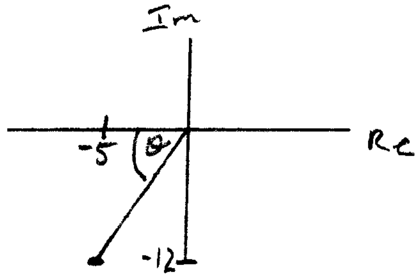
b) $|z^2| = \sqrt{(-5)^2 + (-12^2)}$

$= \sqrt{25+144}$

$= \sqrt{169}$

$= 13$

c)

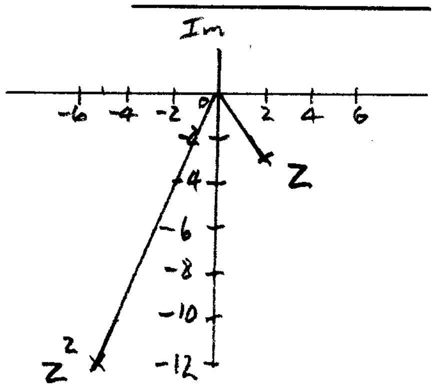


$\arg(z^2) = -\pi + \theta$

$= -\pi + \tan^{-1}\left(\frac{12}{5}\right)$

$= -1.97$ to 2 d.p.

d)



JAN 2011

1) $z = 5 - 3i, \quad w = 2 + 2i$

a) $z^2 = (5-3i)(5-3i)$

$= 25 - 15i - 15i + 9i^2$

$= 25 - 30i - 9$

$= 16 - 30i$

b)

$\frac{z}{w} = \frac{5-3i}{2+2i}$

$= \frac{5-3i}{2+2i} \times \frac{2-2i}{2-2i}$

$= \frac{10 - 6i - 10i + 6i^2}{2^2 + 2^2}$

$= \frac{4 - 16i}{8}$

$= \frac{1}{2} - 2i$

4) $z^2 + pz + q = 0$

a) $2 - 4i$ is a root

$\Rightarrow$ other root is $2 + 4i$

b)

$\alpha + \beta = -p$

$\Rightarrow 2 - 4i + 2 + 4i = -p$

$4 = -p$

$\Rightarrow p = -4$

4b) cont

$\alpha\beta = q$

$\Rightarrow (2-4i)(2+4i) = q$

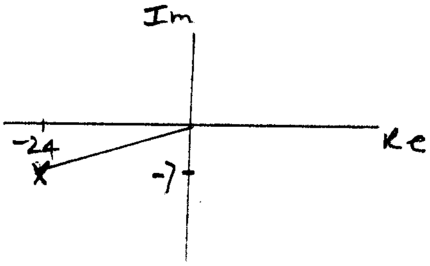
$\Rightarrow 2^2 + 4^2 = q$

$\Rightarrow q = 20$

7)

$z = -24 - 7i$

a)



b) $\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$

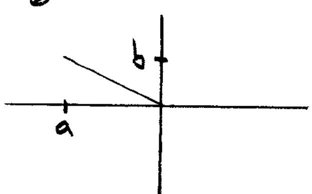
$= -2.86$ to 2d.p.

c) $w = a + bi$

$|w| = 4 \Rightarrow \sqrt{a^2+b^2} = 4$

$\Rightarrow a^2 + b^2 = 16$ (1)

$\arg w = \frac{5\pi}{6}$



$\Rightarrow \tan\frac{\pi}{6} = \frac{b}{-a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{-a}$

$\Rightarrow -a = \sqrt{3}\,b$

$\Rightarrow a^2 = 3b^2$ (2)

From (1) and (2)

$3b^2 + b^2 = 16$

$4b^2 = 16$

$b^2 = 4$

$\Rightarrow b = 2$ since $b > 0$

$a^2 = 3 \times 4 = 12$

$\Rightarrow a = -\sqrt{12}$ since $a < 0$

$a = -2\sqrt{3}$

d) $|zw| = |z| \times |w|$

$= \sqrt{(-24)^2 + (-7)^2} \times 4$

$= 25 \times 4$

$= 100$

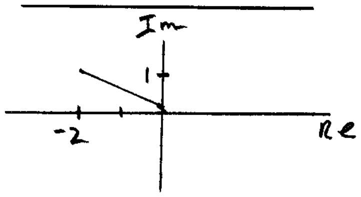
JUN 2011

2) $z_1 = -2 + i$

a) $|z_1| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

or 2.24 to 3 s.f.

b)



$$\arg(z_1) = \pi - \tan^{-1}\left(\frac{1}{2}\right)$$

$$= 2.68 \quad \text{to 2 d.p.}$$

$$z^2 - 10z + 28 = 0$$

c) $$z = \frac{+10 \pm \sqrt{100 - 112}}{2}$$

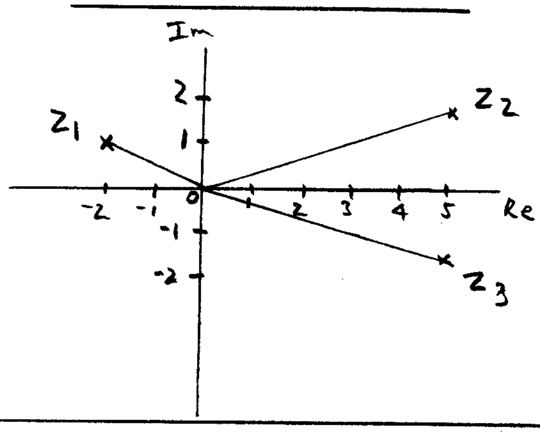
$$z = \frac{+10 \pm \sqrt{12}}{2}$$

$$z = \frac{+10 \pm 2\sqrt{3}\,i}{2}$$

$$z_2 = 5 + \sqrt{3}\,i$$

$$z_3 = 5 - \sqrt{3}\,i$$

d)



6) $z = x + iy$

$$z + 3iz^* = -1 + 13i$$

$$x + iy + 3i(x - iy) = -1 + 13i$$

$$x + iy + 3ix + 3y = -1 + 13i$$

Equating Re and Im parts

$$x + 3y = -1 \quad (1)$$

$$3x + y = 13 \quad (2)$$

(2)×3 $$9x + 3y = 39 \quad (3)$$

(3) − (1) $$8x = 40$$

$$x = \frac{40}{8}$$

$$x = 5$$

Sub for x in (2)

$$3(5) + y = 13$$

$$15 + y = 13$$

$$y = 13 - 15$$

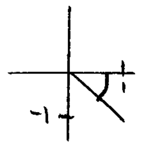
$$y = -2$$

$$\begin{cases} x = 5 \\ y = -2 \end{cases}$$

JAN 2012

1) $z_1 = 1 - i$

a)



$\arg z_1 = -\frac{\pi}{4}$

b) $z_2 = 3 + 4i$

$z_1 z_2 = (1-i)(3+4i)$

$= 3 - 3i + 4i - 4i^2$

$= 7 + i$

c) $\frac{z_2}{z_1} = \frac{3+4i}{1-i}$

$= \frac{3+4i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{3+4i+3i+4i^2}{1^2+1^2}$

$= \frac{-1+7i}{2}$

$= -\frac{1}{2} + \frac{7}{2}i$

5) $z^3 - 8z^2 + 22z - 20 = 0$

a) Given $z_1 = 3 + i$

$z_2 = 3 - i$

Sum of roots $= -\frac{-8}{1} = +8$

$z_1 + z_2 + z_3 = 8$

$3 + i + 3 - i + z_3 = 8$

$6 + z_3 = 8$

$z_3 = 8 - 6$

$z_3 = 2$

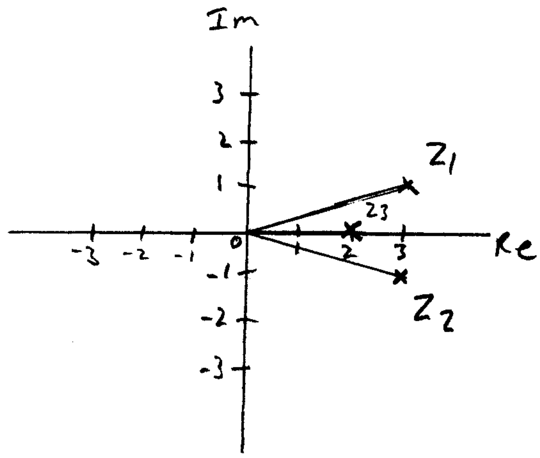
Roots are:

$z_1 = 3 + i$

$z_2 = 3 - i$

$z_3 = 2$

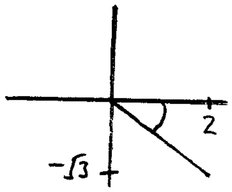
b)



JUN 2012

7) $z = 2 - i\sqrt{3}$

a)



$$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$
$$= -0.71 \text{ to 2 d.p.}$$

b) $$z^2 = (2 - i\sqrt{3})(2 - i\sqrt{3})$$
$$= 4 - i2\sqrt{3} - i2\sqrt{3} + 3i^2$$
$$= 1 - i4\sqrt{3}$$

$\therefore z + z^2$
$$= 2 - i\sqrt{3} + 1 - i4\sqrt{3}$$
$$= 3 - i5\sqrt{3}$$
$$= 3 - 5i\sqrt{3}$$

c) $$\frac{z+7}{z-1} = \frac{9 - i\sqrt{3}}{1 - i\sqrt{3}}$$
$$= \frac{9 - i\sqrt{3}}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}}$$
$$= \frac{9 - i\sqrt{3} + 9i\sqrt{3} - 3i^2}{1^2 + \sqrt{3}^2}$$
$$= \frac{12 + 8i\sqrt{3}}{4}$$
$$= 3 + 2i\sqrt{3}$$

d) $$w = \lambda - 3i$$

$$\arg(4 - 5i + 3w) = -\frac{\pi}{2}$$
$$\Rightarrow \arg(4 - 5i + 3\lambda - 9i) = -\frac{\pi}{2}$$
$$\Rightarrow \arg((4 + 3\lambda) - 14i) = -\frac{\pi}{2}$$
$$\Rightarrow 4 + 3\lambda = 0$$
$$3\lambda = -4$$
$$\lambda = -\frac{4}{3}$$

JAN 2013

2) $$z = \frac{50}{3+4i}$$

a) $$z = \frac{50}{3+4i} \times \frac{3-4i}{3-4i}$$

$$z = \frac{50(3-4i)}{3^2+4^2}$$

$$z = \frac{50(3-4i)}{25}$$

$$z = 6-8i$$

b) $$z^2 = (6-8i)(6-8i)$$

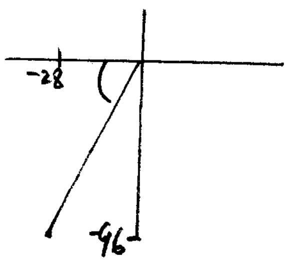
$$= 36 - 48i - 48i + 64i^2$$

$$= -28 - 96i$$

c) $$|z| = \sqrt{6^2+(-8)^2}$$

$$= 10$$

d)



$$\arg(z^2) = -\pi + \tan^{-1}\left(\frac{96}{28}\right)$$

$$= -1.9 \text{ to 1 d.p.}$$

5) a) $$f(x) = (4x^2+9)(x^2-6x+34)$$

Roots when $4x^2+9=0$

$$4x^2 = -9$$

$$x^2 = -\frac{9}{4}$$

$$x = \pm\frac{3}{2}i$$

Roots when $x^2-6x+34=0$

$$x = \frac{+6 \pm \sqrt{36-136}}{2}$$

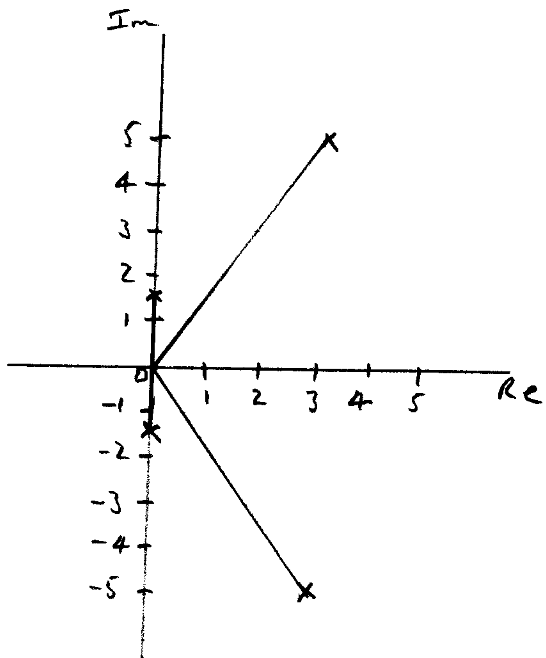
$$x = \frac{+6 \pm 10i}{2}$$

$$x = +3 \pm 5i$$

Roots are

$0+\frac{3}{2}i$, $0-\frac{3}{2}i$, $+3+5i$, $+3-5i$

b)



JUN 2013

7) $z_1 = 2+3i, \quad z_2 = 3+2i$

$z_3 = a+bi$

a) $|z_1+z_2| = |5+5i|$

$= \sqrt{5^2+5^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

b) $w = \dfrac{z_1 z_3}{z_2}$

$w = \dfrac{(2+3i)(a+bi)}{(3+2i)}$

$w = \dfrac{(2+3i)(a+bi)(3-2i)}{(3+2i)(3-2i)}$

$w = \dfrac{(6+9i-4i-6i^2)(a+bi)}{3^2+2^2}$

$w = \dfrac{(12+5i)(a+bi)}{13}$

$w = \dfrac{12a+5ai+12bi-5b}{13}$

$w = \dfrac{12a-5b}{13} + \left(\dfrac{5a+12b}{13}\right)i$

c) Given $w = \dfrac{17}{13} - \dfrac{7}{13}i$

$\Rightarrow \quad 12a - 5b = 17 \quad (1)$

$5a + 12b = -7 \quad (2)$

$(1)\times 5 \quad 60a - 25b = 85 \quad (3)$

$(2)\times 12 \quad 60a + 144b = -84 \quad (4)$

$(4)-(3) \quad 169b = -169$

$b = \dfrac{-169}{169}$

$b = -1$

Sub for b in (1) $\quad 12a + 5 = 17$

$12a = 17-5$

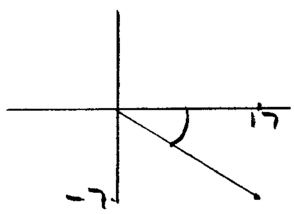
$12a = 12$

$a = 1$

$a = 1, \quad b = -1$

d) $w = \dfrac{1}{13}(17-7i)$

$\arg w = \arg(17-7i)$



$\arg(w) = -\tan^{-1}\left(\dfrac{7}{17}\right)$

$= -0.391$ to 3 d.p

JUN 2014

1) $z_1 = p + 2i, \quad z_2 = 1 - 2i$

a) $$\frac{z_1}{z_2} = \frac{p+2i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{p + 2i + 2pi - 4}{1^2 + 2^2}$$

$$= \frac{p-4}{5} + \frac{(2+2p)}{5}i$$

$$= \frac{p-4}{5} + \frac{2(1+p)}{5}i$$

b) $$\left|\frac{z_1}{z_2}\right| = 13$$

$$\Rightarrow \left(\frac{p-4}{5}\right)^2 + \left(\frac{2(1+p)}{5}\right)^2 = 169$$

$$\Rightarrow (p-4)^2 + 4(1+p)^2 = 169 \times 25$$

$$\Rightarrow p^2 - 8p + 16 + 4(1 + 2p + p^2) = 4225$$

$$p^2 \cancel{-8p} + 16 + 4 \cancel{+8p} + 4p^2 = 4225$$

$$5p^2 = 4205$$

$$p^2 = 841$$

$$p = \pm\sqrt{841}$$

$$p = \pm 29$$

3) $2,\ 1 - 5i$ are roots

a) Other root is $1 + 5i$

b) $$x^3 + px^2 + 30x + q = 0$$

Sum of roots $= -p$

$$2 + 1 - 5i + 1 + 5i = -p$$

$$\Rightarrow p = -4$$

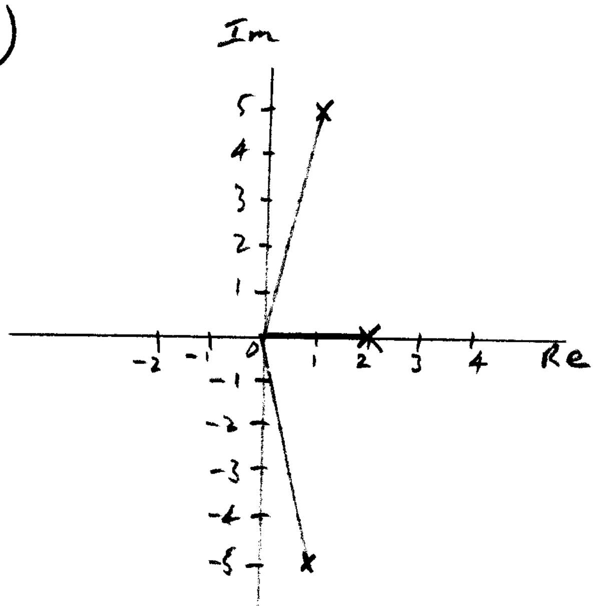
Product of roots $= -q$

$$2(1+5i)(1-5i) = -q$$

$$2(1^2 + 5^2) = -q$$

$$\Rightarrow q = -52$$

c)



JUN 2015

4) $$z_1 = 3i$$

$$z_2 = \frac{6}{1+i\sqrt{3}}$$

a) $$z_2 = \frac{6}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{6-6i\sqrt{3}}{1^2+\sqrt{3}^2}$$

$$= \frac{3}{2} - \frac{3}{2}i\sqrt{3}$$

or $$\frac{3}{2} - i\frac{3\sqrt{3}}{2}$$

b) $$|z_2| = \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{-3\sqrt{3}}{2}\right)^2}$$

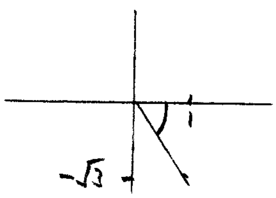
$$= \sqrt{\frac{9}{4}+\frac{27}{4}}$$

$$= \sqrt{\frac{36}{4}}$$

$$= 3$$

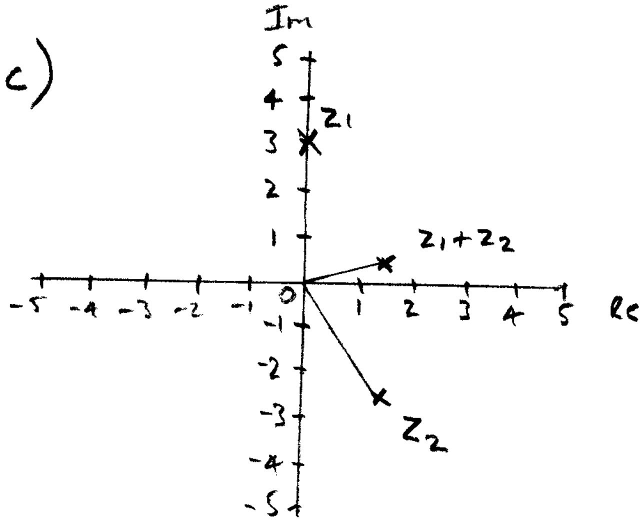
$$z_2 = \frac{3}{2}\left(1 - i\sqrt{3}\right)$$

$$\arg z_2 = \arg\left(1 - i\sqrt{3}\right)$$



$$= -\tan^{-1}\frac{\sqrt{3}}{1} = -\frac{\pi}{3}$$

c)



JUN 2016

4) $$z = \frac{4}{1+i}$$

a) $$z = \frac{4}{1+i} \times \frac{1-i}{1-i}$$

$$z = \frac{4(1-i)}{1^2+1^2}$$

$$z = 2(1-i)$$

$$z = 2 - 2i$$

b) $$z^2 = (2-2i)(2-2i)$$

$$z^2 = 4 - 4i - 4i + 4i^2$$

$$z^2 = -8i$$

c) Roots $2 - 2i$

and $2 + 2i$

$$\alpha + \beta = 4, \quad \alpha\beta = (2-2i)(2+2i) = 2^2 + 2^2 = 8$$

4 cont) Required equation

$$x^2 - 4x + 8 = 0$$

$$\Rightarrow p = -4, \quad q = 8$$

7)

$$z = a + 2i$$

a) $z^2 = (a+2i)(a+2i)$

$$= a^2 + 2ai + 2ai + 4i^2$$

$$= a^2 - 4 + 4ai$$

$\therefore z^2 + 2z = a^2 - 4 + 4ai + 2a + 4i$

$$= (a^2 + 2a - 4) + (4a+4)i$$

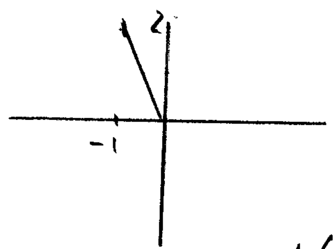
b) $z^2 + 2z$ is real so $4a + 4 = 0$

$$a = -1$$

c) $z = -1 + 2i$

$$|z| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

or 2.24 to 3 s.f.

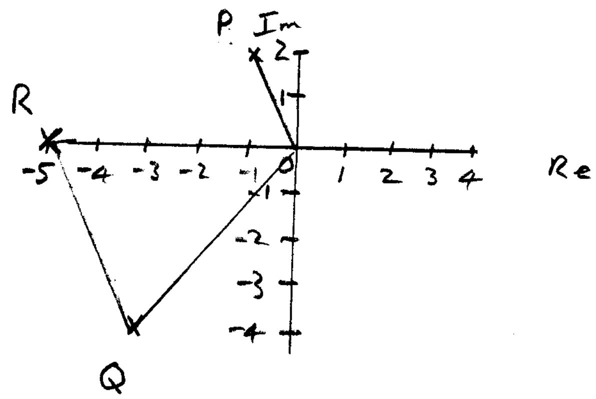


$$\arg z = \pi - \tan^{-1}\left(\frac{2}{1}\right)$$

$$= 2.03 \quad \text{to 3 s.f.}$$

d) $z = -1 + 2i \qquad z^2 = -3 - 4i$

$$z^2 + 2z = -5$$



OP is parallel to QR and half its length

4c) alternative method

$$x^2 + px + q = 0$$

From parts a and b

$$z = 2 - 2i, \quad z^2 = -8i$$

$$\Rightarrow -8i + p(2-2i) + q = 0$$

$$2p + q + i(-8 - 2p) = 0$$

Equating real and imaginary parts

$$-8 - 2p = 0$$

$$-8 = 2p$$

$$-4 = p$$

Also $2p + q = 0$

$$-8 + q = 0$$

$$q = 8$$

Solution:

$$p = -4$$
$$q = 8$$