FM CORE PURE UNIT 5 ALGEBRA + FUNCTIONS

1) $kx^2 + 6kx + 30 = 0$

Roots $p, 3p$

$\Sigma\alpha = p + 3p = 4p$

$\alpha\beta = p \times 3p = 3p^2$

$\Rightarrow 4p = \frac{-6k}{k} = -6$

$\Rightarrow p = -\frac{6}{4} = -\frac{3}{2}$

$p = -\frac{3}{2}$

$3p^2 = \frac{30}{k}$

$3 \times \frac{9}{4} = \frac{30}{k}$

$\frac{27}{4} = \frac{30}{k}$

$27k = 120$

$k = \frac{120}{27}$

$k = \frac{40}{9}$

2) $5x^2 + mx + n = 0$

a) Roots α, α^*

$Re(\alpha) = 4$

$\Rightarrow \alpha + \alpha^* = 8$

$\Rightarrow 8 = -\frac{m}{5}$

$40 = -m$

$m = -40$

b) $Im(\alpha) \neq 0$

$\Rightarrow b^2 - 4ac < 0$

$m^2 - 4 \times 5 \times n < 0$

$1600 - 20n < 0$

$1600 < 20n$

$\frac{1600}{20} < n$

$n > 80$

3) $5z^3 - 11z^2 + kz - 50 = 0$

a) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{k}{5}$

$\alpha\beta\gamma = \frac{50}{5} = 10$

$\alpha + \beta + \gamma = \frac{11}{5}$

b) $\alpha = 1 + 7i$

$\Rightarrow \beta = 1 - 7i$

$\alpha + \beta = 2$

$\alpha + \beta + \gamma = \frac{11}{5}$

3b cont) $\Rightarrow \gamma = \frac{1}{5}$

3c) $(1+7i)\frac{1}{5} + (1-7i)\frac{1}{5} + (1+7i)(1-7i) = \frac{k}{5}$

$\Rightarrow \frac{1}{5} + \frac{1}{5} + 1 + 49 = \frac{k}{5}$

$1 + 1 + 5 + 245 = k$

$k = 252$

4) $f(z) = z^3 + mz^2 + nz - 52$

$\alpha \times \frac{1}{\alpha} \times \left(\alpha + \frac{13}{\alpha} + 46\right) = 52$

$\Rightarrow \alpha + \frac{13}{\alpha} + 46 = 52$

$\alpha + \frac{13}{\alpha} = 6$

$\alpha^2 + 13 = 6\alpha$

$\alpha^2 - 6\alpha + 13 = 0$

By calc
$\alpha = 3+2i$ or $3-2i$

$\alpha = 3+2i$

$\Rightarrow \frac{1}{\alpha} = \frac{3}{13} - \frac{2}{13}i$

but this is not

the same as $3-2i$

the complex conjugate

Conclusion

This question is flawed

and cannot be answered.

5) $4x^4 - 24x^3 + mx^2 - 276x + n = 0$

$\delta = 2+4i$

$\gamma = \delta^* = 2-4i$

a) $\alpha+\beta+\gamma+\delta = -\frac{-24}{4} = 6$

$\gamma+\delta = 2+4i+2-4i = 4$

$\Rightarrow \quad \alpha+\beta = 2$

$\alpha+\beta-2 = 0$

$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

$= -\frac{-276}{4} = 69$

$\Rightarrow \alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = 69$

$\Rightarrow 4\alpha\beta + (\alpha+\beta)(2^2+4^2) = 69$

$4\alpha\beta + 20(\alpha+\beta) = 69$

b) Sub for $\alpha+\beta = 2$

$4\alpha\beta + 20(2) = 69$

$4\alpha\beta + 40 = 69$

$4\alpha\beta = 29$

But $\alpha = 2-\beta$

$\Rightarrow 4(2-\beta)\beta = 29$

$8\beta - 4\beta^2 = 29$

$4\beta^2 - 8\beta + 29 = 0$

$\beta = 1+2.5i$

or $\beta = 1-2.5i$

Roots are

$1+2.5i, \quad 1-2.5i$

$2+4i, \quad 2-4i$

c) $\alpha\beta\gamma\delta = \frac{n}{4}$

$4\alpha\beta\gamma\delta = n$

$(29)(2^2+4^2) = n$

$29 \times 20 = n$

$n = 580$

6) $3x^3 - 6x^2 - 10x - 20 = 0$

a) $\alpha\beta+\beta\gamma+\alpha\gamma = -\frac{10}{3}$

$\alpha\beta\gamma = -\frac{-20}{3} = \frac{20}{3}$

$\alpha+\beta+\gamma = -\frac{-6}{3} = 2$

b) i) $\alpha^4\beta^4\gamma^4 = (\alpha\beta\gamma)^4 = \left(\frac{20}{3}\right)^4$

$= \frac{160000}{81}$

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6b) cont) $\alpha^2+\beta^2+\gamma^2$

ii) $(\alpha+\beta+\gamma)^2$

$= (\alpha+\beta+\gamma)(\alpha+\beta+\gamma)$

$= \alpha^2+\alpha\beta+\alpha\gamma$

$+ \beta^2+\alpha\beta+\beta\gamma$

$+ \gamma^2+\alpha\gamma+\beta\gamma$

$= \alpha^2+\beta^2+\gamma^2+2\Sigma\alpha\beta$

$\therefore \alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2-2\Sigma\alpha\beta$

$= 2^2-2\left(-\frac{10}{3}\right)$

$= 4+\frac{20}{3}$

$= \frac{32}{3}$

iii) $(2-\alpha)(2-\beta)(2-\gamma)$

$= (4-2\alpha-2\beta+\alpha\beta)(2-\gamma)$

$= 8-4\alpha-4\beta+2\alpha\beta$

$-4\gamma+2\alpha\gamma+2\gamma\beta-\alpha\beta\gamma$

$= 8-4\Sigma\alpha+2\Sigma\alpha\beta-\alpha\beta\gamma$

$= 8-4(2)+2\left(-\frac{10}{3}\right)-\frac{20}{3}$

$= 8-8-\frac{20}{3}-\frac{20}{3}$

$= -\frac{40}{3}$

7) $2x^3-4x^2+6x-9=0$

Let $w = 2x-1$

$w+1 = 2x$

$\frac{w+1}{2} = x$

Required eqn

$2\left(\frac{w+1}{2}\right)^3-4\left(\frac{w+1}{2}\right)^2+6\left(\frac{w+1}{2}\right)-9=0$

$\frac{2(w^3+3w^2+3w+1)}{8}-\frac{4(w^2+2w+1)}{4}$

$+\frac{6(w+1)}{2}-9=0$

$\times 4$

$1(w^3+3w^2+3w+1)-4(w^2+2w+1)$

$+12(w+1)-36=0$

w^3+3w^2+3w+1
$-4w^2-8w-4$
$+12w+12$
$-36=0$

$w^3-w^2+7w-27=0$

8) $2x^4 - 6x^2 + 16x - 1 = 0$

$2x^4 + 0x^3 - 6x^2 + 16x - 1 = 0$

Let $w = 2x$

$\frac{w}{2} = x$

Eqn required is

$2\left(\frac{w}{2}\right)^4 - 6\left(\frac{w}{2}\right)^2 + 16\left(\frac{w}{2}\right) - 1 = 0$

$\frac{w^4}{8} - \frac{3w^2}{2} + 8w - 1 = 0$

$w^4 - 12w^2 + 64w - 8 = 0$

$p = 1$

$q = 0$

$r = -12$

$s = 64$

$t = -8$