

Moments about 0

$$12g \bar{x} = 0.8 \times 3g + 1 \times 4g + 1.4 \times 5g$$

$$12 \bar{x} = 2.4 + 4 + 7$$

$$\bar{x} = \frac{13.4}{12} = \frac{67}{60} \text{ m from 0}$$

b)

$$(12+m) \times 1.5 = \frac{67}{60} \times 12 + mx$$

$$18 + 1.5m = \frac{67}{5} + mx$$

$$18 - \frac{67}{5} = mx - 1.5m$$

$$4.6 = m(x - 1.5)$$

$$m = \frac{4.6}{x - 1.5}$$

c) Max  $x = 2\text{m}$  (length of rod)

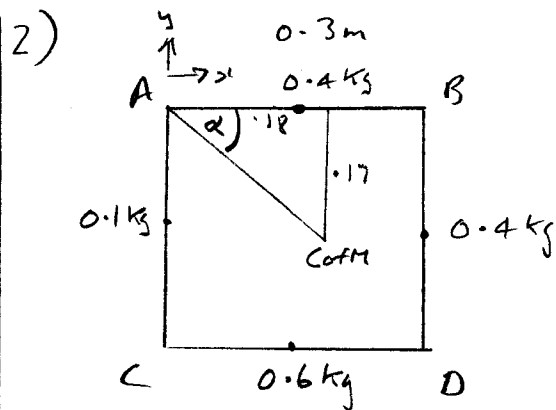
$$\text{Then } m = \frac{4.6}{2 - 1.5} = \frac{4.6}{0.5}$$

$$= 9.2 \text{ kg}$$

For  $x < 2\text{m}$

denominator  $x - 1.5$  would be smaller so  $m$  would be bigger

$$\therefore m \geq 9.2 \text{ kg}$$



Let A be origin

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} \times 1.5 = \begin{pmatrix} 0.15 \\ 0 \end{pmatrix} \times 0.4$$

$$+ \begin{pmatrix} 0.3 \\ -0.15 \end{pmatrix} \times 0.4$$

$$+ \begin{pmatrix} 0.15 \\ -0.3 \end{pmatrix} \times 0.6$$

$$+ \begin{pmatrix} 0 \\ -0.15 \end{pmatrix} \times 0.1$$

$$1.5 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 0.06 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.12 \\ -0.06 \end{pmatrix} + \begin{pmatrix} 0.09 \\ -0.18 \end{pmatrix} + \begin{pmatrix} 0 \\ -0.015 \end{pmatrix}$$

$$1.5 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 0.27 \\ -0.255 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 0.18 \\ -0.17 \end{pmatrix}$$

AB makes angle of  $\alpha$  with vertical

$$\alpha = \tan^{-1} \left( \frac{0.17}{0.18} \right)$$

$$\alpha = 43.4^\circ$$

3)



$$60^\circ = 2\alpha$$

$$\frac{\pi}{6} = \alpha$$

Arc com  $\frac{r \sin \alpha}{\alpha}$  from centre

$$= \frac{15 \sin \frac{\pi}{6}}{\frac{\pi}{6}}$$

$$= \frac{45}{\pi} \text{ cm}$$

Sector com  $\frac{2r \sin \alpha}{3\alpha}$  from centre

$$= \frac{2 \times 15 \times \sin \frac{\pi}{6}}{\frac{3\pi}{6}}$$

$$= \frac{30}{\pi} \text{ cm}$$

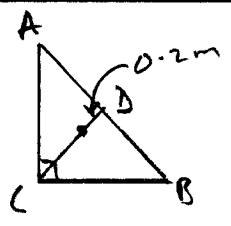
$$\bar{x} \times 600 = \frac{45}{\pi} \times 150 + \frac{30}{\pi} \times 450$$

$$600 \bar{x} = \frac{20250}{\pi}$$

$$\bar{x} = \frac{20250}{600\pi}$$

$$\bar{x} = 10.7 \text{ cm from centre}$$

4)  
a)



com is  $\frac{2}{3}$  along median from vertex

$$\Rightarrow \text{median} = 3 \times 0.2 = 0.6 \text{ m}$$

If  $CD = 0.6 \text{ m}$

$$\Rightarrow AD = DB = 0.6 \text{ m also (isos } \Delta \text{)}$$

Area of  $\Delta = \frac{1}{2} AB \times DC$

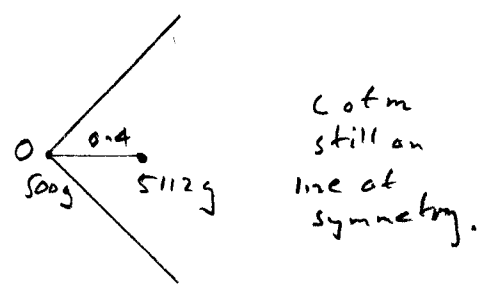
$$= \frac{1}{2} \times 1.2 \times 0.6$$

$$= 0.36 \text{ m}^2$$

b) Mass of lamina

$$0.36 \times 100^2 \times 1.42 \text{ g}$$

$$= 5112 \text{ g}$$



$$\bar{x} \times (5112 + 500) = 0 \times 500 + 0.4 \times 5112$$

$$5612 \bar{x} = 2044.8$$

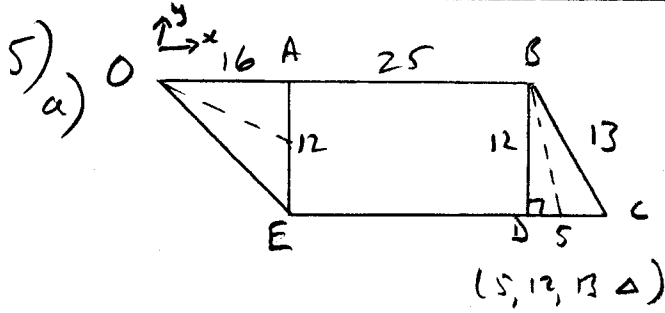
$$\bar{x} = \frac{2044.8}{5612}$$

$$\bar{x} = 0.364 \text{ m}$$

Distance moved =  $0.4 - 0.364$

$$= 0.0356 \text{ m}$$

$$= 36 \text{ mm to next mm}$$



CoM of  $\triangle OAE$

$$x = \frac{2}{3} \times 16 = \left( \frac{32}{3} \right)$$

$$y = \frac{2}{3} \times 0 - 6 = -4$$

CoM of  $\square ABDE$

$$x = 16 + 12.5 = (28.5)$$

$$y = -6$$

CoM of  $\triangle BCD$

$$x = 16 + 25 + \frac{2}{3} \times 5 = \left( \frac{133}{3} \right)$$

$$y = \frac{2}{3} \times 0 - 12 = -8$$

Area of  $\triangle OAE = 6 \times 16 = 96 \text{ cm}^2$

Area of  $\square ABDE = 25 \times 12 = 300 \text{ cm}^2$

Area of  $\triangle BCD = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

$$(96 + 300 + 30) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$= 96 \begin{pmatrix} \frac{32}{3} \\ -4 \end{pmatrix} + 300 \begin{pmatrix} 28.5 \\ -6 \end{pmatrix} + 30 \begin{pmatrix} \frac{133}{3} \\ -8 \end{pmatrix}$$

$$426 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 10904 \\ -2424 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 25.60 \\ -5.69 \end{pmatrix} \text{ cm}$$

5b) He wants CoM to be at  $\begin{pmatrix} 23 \\ -6 \end{pmatrix}$

which is midpoint of long diagonal

Mass lost to hole =  $\pi r^2$   
 $= 16\pi$

Require

$$(426 - 16\pi) \begin{pmatrix} 23 \\ -6 \end{pmatrix}$$

$$= 426 \begin{pmatrix} 25.60 \\ -5.69 \end{pmatrix} - 16\pi \begin{pmatrix} x \\ y \end{pmatrix}$$

where  $(x, y)$  is centre of circle to be removed.

$$16\pi \begin{pmatrix} x \\ y \end{pmatrix} = 426 \begin{pmatrix} 25.60 \\ -5.69 \end{pmatrix} - (426 - 16\pi) \begin{pmatrix} 23 \\ -6 \end{pmatrix}$$

$$16\pi \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2263.706 \\ -169.533 \end{pmatrix}$$

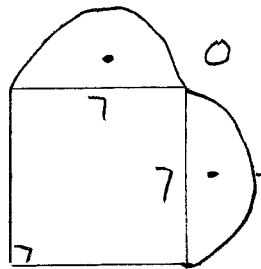
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 45.03 \\ -3.37 \end{pmatrix}$$

5b) This is not possible as the 4cm radius circle centre (45.03, -3.37) would extend outside the trapezium.

For example, the highest point of the circle would have y coord  $-3.37 + 4 = +0.63$  cm

yet there is no point on the trapezium with a positive y-coordinate.

6)



$r = 3.5$  cm

For sector  $d = \frac{2r \sin \alpha}{3\alpha}$

$\alpha = \frac{\pi}{2}$   
 $d = \frac{2 \times 3.5 \sin \frac{\pi}{2}}{3 \times \frac{\pi}{2}}$

$d = 1.4854$

CoM of top arc  $(-3.5, 1.485)$

CoM of side arc  $(1.485, -3.5)$

CoM of square  $(-3.5, -3.5)$

$\left(\frac{\bar{x}}{\bar{y}}\right) \times \left(7^2 + \pi \times 3.5^2\right)$

$= \frac{\pi \times 3.5^2}{2} \begin{pmatrix} -3.5 \\ 1.485 \end{pmatrix} + \frac{\pi \times 3.5^2}{2} \begin{pmatrix} 1.485 \\ -3.5 \end{pmatrix} + 49 \begin{pmatrix} -3.5 \\ -3.5 \end{pmatrix}$

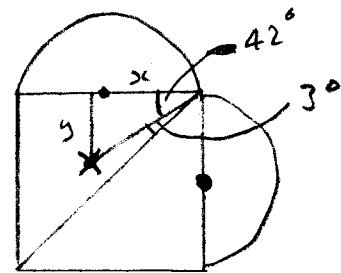
$87.4845 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} -210.273 \\ -210.273 \end{pmatrix}$

$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} -2.4035 \\ -2.4035 \end{pmatrix}$  cm

6b)  $(1.8 + 3m + 4m) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

$= 1.8 \begin{pmatrix} -2.403 \\ -2.403 \end{pmatrix} + 4m \begin{pmatrix} -3.5 \\ 0 \end{pmatrix} + 3m \begin{pmatrix} 0 \\ -3.5 \end{pmatrix}$

$(1.8 + 7m) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} -4.3254 - 14m \\ -4.3254 - 10.5m \end{pmatrix}$



$\tan 42^\circ = \frac{15}{12}$

$\tan 42^\circ = \frac{4.3254 + 10.5m}{1.8 + 7m}$   
 $\frac{4.3254 + 14m}{1.8 + 7m}$

$\tan 42^\circ = \frac{4.3254 + 10.5m}{4.3254 + 14m}$

6b)  
cont)

$$(4.3254 + 14m) \tan 42^\circ = 4.3254 + 10.5m$$

$$4.3254 \tan 42^\circ + 14m \tan 42^\circ = 4.3254 + 10.5m$$

$$m(14 \tan 42^\circ - 10.5) = 4.3254 - 4.3254 \tan 42^\circ$$

$$m = \frac{4.3254(1 - \tan 42^\circ)}{14 \tan 42^\circ - 10.5}$$

$$m = 0.2046$$

$$m = 0.2 \text{ g}$$

$$\text{Diamond} = 3m = 0.6 \text{ g}$$

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