1 Given that $\mathbf{M} = \begin{pmatrix} 2 & p+4 \\ -p & 3 \end{pmatrix}$, show that $\mathbf{M}$ is non-singular for all values of p.

(3 marks)

2 The matrix $\mathbf{M} = \begin{pmatrix} a & 21 \\ b & -8 \end{pmatrix}$ is such that $\mathbf{M}^2 = \mathbf{I}$. Find the values of a and b.

(3 marks)

3 $\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$

a Write down the transformation represented by matrix $\mathbf{M}$.

(2 marks)

b The transformation represented by $\mathbf{M}$ maps the line segment with endpoints $P(1, 2)$ and $Q(2, 5)$ onto a line segment $P'Q'$.

Find the coordinates of P' and Q'.

(3 marks)

c Find $\mathbf{M}^2$ and describe the transformation it represents.

(3 marks)

4 $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

a Write down fully the transformation represented by matrix $\mathbf{A}$.

(1 mark)

b A point (a, b) is transformed onto the point $(4+2a, 5+b)$ by matrix $\mathbf{A}$. Find the values of a and b.

(3 marks)

5 $\mathbf{P} = \begin{pmatrix} q & \sqrt{5} \\ \sqrt{5} & -q \end{pmatrix}$, where q is a constant.

A triangle, T, is transformed using matrix $\mathbf{P}^2$. Describe fully the transformation represented by $\mathbf{P}^2$.

(3 marks)

6 **a** State the 3 × 3 matrix, **M**, representing a reflection in the plane $y = 0$. **(1 mark)**

A line segment from the point (2, 5, 9) to the point (8, 5, 9) is transformed using the matrix **M**.

b Find the coordinates of the image of the midpoint of the line segment.

(3 marks)

c Find the matrix, **N**, required to transform the image back to the original position of the line segment.

(2 marks)

7 $\mathbf{M} = \begin{pmatrix} -1 & -\sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$

a Show that **M** is non-singular.

(2 marks)

The rhombus *R* is transformed to the rhombus *S* by the transformation represented by the matrix **M**. Given that the rhombus *R* has an area of 20 square units,

b find the area of rhombus *S*.

(1 mark)

The matrix **M** represents an enlargement, centre *O* and scale factor *k*, where $k > 0$, followed by a rotation though angle θ anticlockwise about *O*.

c Find the value of *k*.

(2 marks)

d Find the value of θ.

(2 marks)

8 $\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

a Find the transformation represented by matrix **M**.

(2 marks)

b The point $P(-p, 0, p)$ is transformed using matrix **M**. Find, in terms of *p*, the exact coordinates of the image of this point.

(2 marks)

9 In the year 2001, a parliamentary constituency had an area of 200 square kilometres. The land was assigned for residential, commercial and recreation. Originally there were 20 more square kilometres assigned to recreation than to commercial. In 2011, following a boundary change, the total area of the parliamentary constituency had increased by 40 square kilometres. In the same time:

- residential area increased by 22%
- commercial area decreased by 10%

- recreational area increased by 28%.

Form and solve a matrix equation to find out the area of land assigned to residential, commercial and recreation in 2001.

(7 marks)

10 $\mathbf{M} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

a Describe fully the geometrical transformation represented by **M**.

(2 marks)

b The point (a, b) is mapped onto $\left(-\sqrt{2}, 3\sqrt{2}\right)$ under the transformation represented by **M**. Find the values of a and b.

(3 marks)