1 The complex numbers w and z are given by $w = 2 + k\text{i}$ and $z = -2k + 4\text{i}$, where k is a real constant. Given that $\arg(w+z) = \frac{3\pi}{4}$, find the exact value of k.

(4 marks)

2 $z = -6 - 6\sqrt{3}\text{i}$

a Express z in the form $r(\cos\theta + \text{i}\sin\theta)$, $-\pi < \theta < \pi$

(3 marks)

b Given that $|w| = 4$ and $\arg w = -\frac{\pi}{2}$, express $\frac{z}{w}$ in the form $r(\cos\theta + \text{i}\sin\theta)$, $-\pi < \theta < \pi$

(3 marks)

3 $|z - 4 + 2\text{i}| = |z + 8 - 6\text{i}|$

a Find the Cartesian equation of this locus.

(4 marks)

b Sketch the locus of z. Label the points of intersection with the real and imaginary axes.

(2 marks)

c Find the exact least possible value of $|z|$. Leave your answer in the form $a\sqrt{13}$, where a is a rational number.

(3 marks)

4 A complex number z is represented by the point Q on the Argand diagram. Given that $|z - 6 - \text{i}| = 5$,

a sketch the locus of Q.

(2 marks)

b Show that the minimum angle of $\arg(z - 11 - 10\text{i})$ in the interval $-\pi < \theta < \pi$ is

$$-\frac{\pi}{2} - 2\arcsin\left(\frac{5}{\sqrt{106}}\right)$$

(5 marks)

5 The region R in an Argand diagram is satisfied by the inequalities $|z-1-3\mathrm{i}| < 3$ and $|z+2+2\mathrm{i}| < |z+2-4\mathrm{i}|$. Draw an Argand diagram and shade in the region R.

(6 marks)

6 Shade on an Argand diagram the set of points

$\{z \in \quad : |z+3-5\mathrm{i}| < 3\} \quad \left\{z \in \quad : -\frac{\pi}{4} < \arg(z+6-5\mathrm{i}) < \frac{\pi}{4}\right\}$

(6 marks)