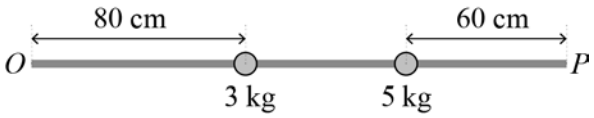
1 Figure 1 shows a uniform rod OP, of length 2 m and mass 4 kg.

A mass of 3 kg is attached to the rod 80 cm from end O.

A mass of 5 kg is attached to the rod 60 cm from end P.

Figure 1



a Find the distance of the centre of mass of this system from O, giving your answer as an exact fraction. **(3 marks)**

b A third mass, m, is added to the rod at a distance of x m from O.

The centre of mass is now 1.5 m from O.

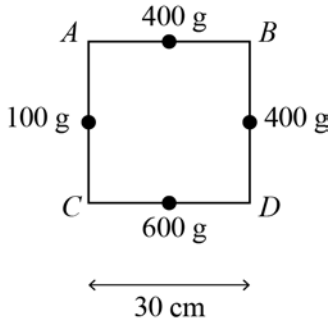
Show that the third mass, m, can be expressed as $m = \dfrac{4.6}{x - 1.5}$ **(3 marks)**

c Explain why $m \geqslant 9.2$ kg. **(2 marks)**

2 A square-framework structure, with corners $ABCD$, is made from identical light, uniform rods, each 30 cm long.

Attached to the centres of the four rods are different masses, as shown in Figure 2.

Figure 2



This frame is suspended freely from point A in its equilibrium position.

Find the angle that the side AB makes with the vertical, to 3 significant figures. **(5 marks)**

3 A lamina is created in the shape of a sector of radius 15 cm which subtends an angle of 60° at the centre of the circle, *O*, from which it is formed.

This lamina has a mass of 450 g.

Plastic edging is attached along the curved edge of this sector-shaped lamina (its arc).

The total mass of the plastic edging is 150 g.

Find the centre of mass of this system, relative to the centre point *O*. **(5 marks)**

4 A right-angled, isosceles, uniform, triangular lamina, has its centre of mass 0.2 m along its line of symmetry, measured from the centre of its longest side.

a Show that the area of the surface of this triangular lamina is 0.36 m^2. **(3 marks)**

b This lamina is made of uPVC with a surface density of 1.42 g cm^{-2}.

A mass of 500 g is now placed on the lamina at its right-angled corner.

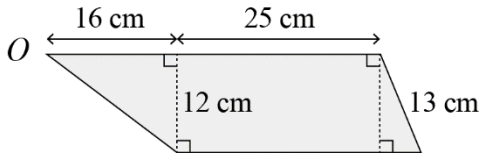
Find the distance, to the nearest millimetre, that the centre of mass moves from its position before the 500 g mass was added. **(3 marks)**

5 Colin cuts an irregular trapezium from a sheet of steel with uniform thickness.

He wants to locate the trapezium's centre of mass.

Its dimensions are shown in Figure 3.

Figure 3



a Find the position of the centre of mass, relative to point *O*.

Take to the right and upwards as the positive *x* and *y* directions respectively. **(8 marks)**

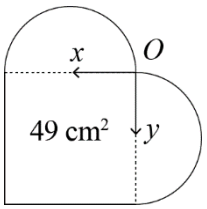
b Colin is unhappy – he wants the centre of mass to lie at the midpoint of the longest diagonal within his shape. He hopes to achieve this by drilling out a circular hole, of radius 4 cm, centred at some point on the surface of the trapezium.

Explain why Colin cannot do this. **(6 marks)**

6 Figure 4 shows a heart-shaped pendant constructed from gold. It has uniform thickness.

The shape is formed by a square of area 49 cm^2, with two semi-circles attached to adjacent sides.

Figure 4



The total mass of the pendant is 1.8 g.

a Find the position of the centre of mass, relative to *O*. **(5 marks)**

b The jeweller mounts a ruby on the pendant's surface, midway along the horizontal diameter of the top semi-circle.

The jeweller then mounts a diamond midway along the vertical diameter of the other semi-circle.

The mass of the ruby is 4*m* grams and the mass of the diamond is 3*m* grams.

When the pendant is suspended freely from the point where the two semi-circles meet, the line of symmetry of the shape makes an angle of 3° to the vertical.

Find the mass of the diamond, *m*, to the nearest tenth of a gram. **(7 marks)**