Name: ______________________________________________

# Exponential and Log Equations

**Date:**

**Time:**

**Total marks available:** 75

**Total marks achieved:** ______

Solutions

## Questions

**Q1.**

Given that

$$2\log_2 (x + 15) - \log_2 x = 6$$

(a) Show that

$x^2 - 34x + 225 = 0$

(5)

(b) Hence, or otherwise, solve the equation

$2\log_2(x + 15) - \log_2 x = 6$

(2)

**(Total 7 marks)**

a) $2\log_2(x+15) - \log_2 x = 6$

$\log_2 (x+15)^2 - \log_2 x = 6$

$\log_2\left(\frac{(x+15)^2}{x}\right) = 6$

$\frac{(x+15)^2}{x} = 2^6$

$(x+15)^2 = 64x$

$x^2 + 30x + 225 = 64x$

$x^2 - 34x + 225 = 0$

b) $(x-9)(x-25) = 0$

$\Rightarrow x = 9$ or $x = 25$

**Q2.**

(a) Find the value of $y$ such that

$$\log_2 y = -3$$

(2)

(b) Find the values of $x$ such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

**(Total 7 marks)**

a) $\log_2 y = -3$

$y = 2^{-3}$

$y = \frac{1}{8}$

b) $\dfrac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$

$2^5 = 32$

$2^4 = 16$

$5 + 4 = (\log_2 x)^2$

$9 = (\log_2 x)^2$

$\pm 3 = \log_2 x$

$\Rightarrow x = 2^3 \quad \text{or} \quad x = 2^{-3}$

$x = 8 \quad \text{or} \quad x = \frac{1}{8}$

**Q3.**

(a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Solve the equation

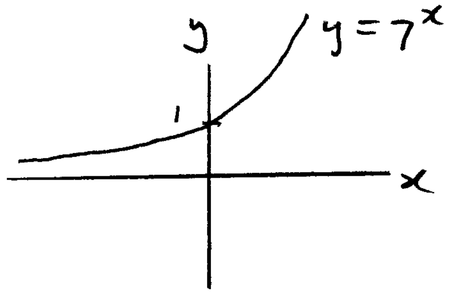
$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)

**(Total 8 marks)**

a)




b)

$$7^{2x} - 4(7^x) + 3 = 0$$

$$(7^x - 1)(7^x - 3) = 0$$

$$\Rightarrow 7^x = 1 \quad \text{or} \quad 7^x = 3$$

$$\Rightarrow x = 0$$

$$\ln 7^x = \ln 3$$

$$x \ln 7 = \ln 3$$

$$x = \frac{\ln 3}{\ln 7}$$

$x = 0.56$ to 2 d.p.

**Q4.**

Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$1 + 2\log_3 x = \log_3(28x - 9)$

(3)

**(Total 6 marks)**

a) $y = 3x^2$

$\log_3 y = \log_3(3x^2)$

$\log_3 y = \log_3 3 + \log_3 x^2$

$\log_3 y = 1 + 2\log_3 x$

b)

$1 + 2\log_3 x = \log_3(28x - 9)$

$\log_3(3x^2) = \log_3(28x - 9)$

$\Rightarrow \quad 3x^2 = 28x - 9$

$3x^2 - 28x + 9 = 0$

$(3x - 1)(x - 9) = 0$

$\Rightarrow x = \frac{1}{3}$ or $x = 9$

**Q5.**

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where $a$ is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$ (2)

(b) factorise $f(x)$ completely. (4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

(c) find the values of $y$ that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate. (3)

**(Total 7 marks)**

a) factor theorem

$(x-3)$ a factor $\Rightarrow f(3)=0$

$f(3) = 2(3)^3 - 5(3)^2 + 3a + 18$

$0 = 54 - 45 + 3a + 18$

$0 = 27 + 3a$

$-27 = 3a$

$-9 = a$

$a = -9$

b)

$$\begin{array}{r} 2x^2 + x - 6 \\ (x-3)\overline{)\,2x^3 - 5x^2 - 9x + 18} \\ \underline{2x^3 - 6x^2} \\ +x^2 - 9x \\ \underline{+x^2 - 3x} \\ -6x + 18 \\ \underline{-6x + 18} \end{array}$$

$f(x) = (x-3)(2x^2+x-6)$

$f(x) = (x-3)(2x-3)(x+2)$

c) $g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$

$g(y) = (3^y - 3)(2\times3^y - 3)(3^y + 2)$

$g(y) = 0$

$\Rightarrow (3^y - 3) = 0$ or $2(3^y) - 3 = 0$

$3^y = 3$

$\Rightarrow y = 1$

$2(3^y) = 3$

$3^y = \frac{3}{2}$

$\ln(3^y) = \ln 1.5$

$y \ln 3 = \ln 1.5$

$y = \frac{\ln 1.5}{\ln 3}$

$y = 0.37$ to 2 d.p.

**Q6.**

(i) Find the exact value of $x$ for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express $y$ in terms of $a$.
Give your answer in its simplest form.

(3)

**(Total 7 marks)**

i) $\log_2(2x) = \log_2(5x+4) - 3$

$\log_2(2x) - \log_2(5x+4) = -3$

$\log_2\left(\frac{2x}{5x+4}\right) = -3$

$\frac{2x}{5x+4} = 2^{-3}$

$\frac{2x}{5x+4} = \frac{1}{8}$

$16x = 5x + 4$

$16x - 5x = 4$

$11x = 4$

$x = \frac{4}{11}$

ii) $\log_a y + 3\log_a 2 = 5$

$\log_a y + \log_a 2^3 = 5$

$\log_a y + \log_a 8 = 5$

$\log_a (8y) = 5$

$8y = a^5$

$y = \frac{a^5}{8}$

**Q7.**

Given that $\log_3 x = a$, find in terms of $a$,

(a) $\log_3 (9x)$

(2)

(b) $\log_3\left(\dfrac{x^5}{81}\right)$

(3)

giving each answer in its simplest form.

(c) Solve, for $x$,

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

(4)

**(Total 9 marks)**

a) $\log_3 x = a$

$\log_3(9x) = \log_3 9 + \log_3 x$

$= 2 + a$

b) $\log_3\left(\dfrac{x^5}{81}\right)$

$= \log_3(x^5) - \log_3(81)$

$= 5\log_3 x - 4$

$= 5a - 4$

c) $\log_3(9x) + \log_3\left(\dfrac{x^5}{81}\right) = 3$

$2 + a + 5a - 4 = 3$

$6a - 2 = 3$

$6a = 3 + 2$

$6a = 5$

$a = \dfrac{5}{6}$

$\Rightarrow \log_3 x = \dfrac{5}{6}$

$x = 3^{5/6}$

$x = 2.498$

to 4 s.f

**Q8.**

(a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

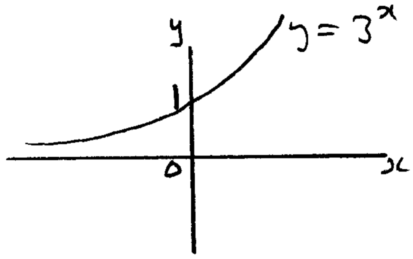
(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)

a)




**(Total 7 marks)**

b) $3^{2x} - 9(3^x) + 18 = 0$

$(3^x - 3)(3^x - 6) = 0$

$\Rightarrow 3^x - 3 = 0$ or $3^x - 6 = 0$

$\Rightarrow 3^x = 3$

$\Rightarrow x = 1$

$3^x = 6$

$\ln 3^x = \ln 6$

$x \ln 3 = \ln 6$

$x = \dfrac{\ln 6}{\ln 3}$

$x = 1.63$ to 2 d.p.

**Q9.**

(i) Solve

$$5^y = 8$$

giving your answer to 3 significant figures.

(2)

(ii) Use algebra to find the values of $x$ for which

$$\log_2(x + 15) - 4 = \tfrac{1}{2} \log_2 x$$

(6)

**(Total 8 marks)**

i)

$$5^y = 8$$

$$\ln(5^y) = \ln 8$$

$$y \ln 5 = \ln 8$$

$$y = \frac{\ln 8}{\ln 5}$$

$y = 1.29$ to 3 s.f.

ii)

$$\log_2(x+15) - 4 = \tfrac{1}{2}\log_2 x$$

$$\log_2(x+15) - \tfrac{1}{2}\log_2 x = 4$$

$$\log_2(x+15) - \log_2 x^{\frac{1}{2}} = 4$$

$$\log_2\left(\frac{x+15}{\sqrt{x}}\right) = 4$$

$$\frac{x+15}{\sqrt{x}} = 2^4$$

$$x + 15 = 16\sqrt{x}$$

square

$$x^2 + 30x + 225 = 256x$$

$$x^2 - 226x + 225 = 0$$

$$(x-1)(x-225) = 0$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = 225$$

**Q10.**

(i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of $y$ such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

**(Total for question = 9 marks)**

i)

$$8^{2x+1} = 24$$

$$\ln\left(8^{2x+1}\right) = \ln 24$$

$$(2x+1)\ln 8 = \ln 24$$

$$2x+1 = \frac{\ln 24}{\ln 8}$$

$$2x = \frac{\ln 24}{\ln 8} - 1$$

$$x = \frac{\frac{\ln 24}{\ln 8} - 1}{2}$$

$x = 0.264$ to 3 d.p.

ii)

$$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1$$

$$\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$$

$$\log_2\left(\frac{11y-3}{3y^2}\right) = 1$$

$$\frac{11y-3}{3y^2} = 2^1$$

$$11y - 3 = 2 \times 3y^2$$

$$0 = 6y^2 - 11y + 3$$

$$0 = (3y-1)(2y-3)$$

$\Rightarrow 3y - 1 = 0$ or $2y - 3 = 0$

$3y = 1$ $\qquad 2y = 3$

$y = \frac{1}{3}$ $\qquad y = \frac{3}{2}$