EDEXCEL FP1 - PROOF BY INDUCTION

Jan 2010

3. A sequence of numbers is defined by

$$u_1 = 2,$$
$$u_{n+1} = 5u_n - 4, \qquad n \geqslant 1.$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 5^{n-1} + 1$.

(4)

Jun 2010

7. $$f(n) = 2^n + 6^n$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

Jan 2011

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$ is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}\left(4^n - 1\right)$$

(5)

EDEXCEL FP1 - PROOF BY INDUCTION

Jun 2011

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$,

(6)

(b) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

Jan 2012

6. (a) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$$

(5)

(b) Using the result in part (a), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4} n(n^3 + 2n^2 + n - 8)$$

(3)

(c) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geqslant 1, \quad u_1 = 1$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that $u_n = 2^n - 1$

(5)

EDEXCEL FP1 - PROOF BY INDUCTION

Jun 2012

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by } 5.$$

(6)

Jan 2013

8. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+3) = \tfrac{1}{3}n(n+1)(n+5)$$

(6)

(b) A sequence of positive integers is defined by

$$u_1 = 1,$$
$$u_{n+1} = u_n + n(3n+1), \qquad n \in \mathbb{Z}^+$$

Prove by induction that

$$u_n = n^2(n-1)+1, \qquad n \in \mathbb{Z}^+$$

(5)

Jun 2013

9. (a) A sequence of numbers is defined by

$$u_1 = 8$$
$$u_{n+1} = 4u_n - 9n, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

EDEXCEL FP1 - PROOF BY INDUCTION

Jun 2014

9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)

Jun 2015

6. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}\left(5^n - 1\right) & 5^n \end{pmatrix}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n\left(4n^2 - 1\right)$$

(6)

Jun 2016

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}$$

(5)

(ii) A sequence of positive rational numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = \frac{1}{3}u_n + \frac{8}{9}, \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 5 \times \left(\frac{1}{3}\right)^n + \frac{4}{3}$$

(5)