EDEXCEL FP1 - MATRICES

Jan 2010

5. $\mathbf{A} = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix}$, where a is real.

(a) Find det **A** in terms of a.

(2)

(b) Show that the matrix **A** is non-singular for all values of a.

(3)

Given that $a = 0$,

(c) find $\mathbf{A}^{-1}$.

(3)

9. $$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the geometrical transformation represented by the matrix **M**.

(2)

The transformation represented by **M** maps the point A with coordinates (p, q) onto the point B with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(b) Find the value of p and the value of q.

(4)

(c) Find, in its simplest surd form, the length OA, where O is the origin.

(2)

(d) Find $\mathbf{M}^2$.

(2)

The point B is mapped onto the point C by the transformation represented by $\mathbf{M}^2$.

(e) Find the coordinates of C.

(2)

EDEXCEL FP1 - MATRICES

Jun 2010

2. $\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}$, where a is a real constant.

(a) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(b) Find the values of a for which $\mathbf{M}$ is singular.

(2)

6. Write down the 2×2 matrix that represents

(a) an enlargement with centre $(0, 0)$ and scale factor 8,

(1)

(b) a reflection in the x-axis.

(1)

Hence, or otherwise,

(c) find the matrix $\mathbf{T}$ that represents an enlargement with centre $(0, 0)$ and scale factor 8, followed by a reflection in the x-axis.

(2)

$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}$, where k and c are constants.

(d) Find $\mathbf{AB}$.

(3)

Given that $\mathbf{AB}$ represents the same transformation as $\mathbf{T}$,

(e) find the value of k and the value of c.

(2)

EDEXCEL FP1 - MATRICES

Jan 2011

2.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**.

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**,

(2)

(c) write down $\mathbf{C}^{100}$.

(1)

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find det **A**.

(1)

(b) Find $\mathbf{A}^{-1}$.

(2)

The triangle R is transformed to the triangle S by the matrix **A**.
Given that the area of triangle S is 72 square units,

(c) find the area of triangle R.

(2)

The triangle S has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of R.

(4)

Jun 2011

3. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

describe fully the geometrical transformation represented by **B**.

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

5. $\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.

Given that the matrix **A** maps the point with coordinates $(4, 6)$ onto the point with coordinates $(2, -8)$,

(a) find the value of a and the value of b.

(4)

A quadrilateral R has area 30 square units.
It is transformed into another quadrilateral S by the matrix **A**.
Using your values of a and b,

(b) find the area of quadrilateral S.

(4)

Jan 2012

4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(a) Find the coordinates of the vertices of T'.

(2)

(b) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(c) Find **QR**.

(2)

(d) Find the determinant of **QR**.

(2)

(e) Using your answer to part (d), find the area of T''.

(3)

8. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular.

(2)

(b) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

EDEXCEL FP1 - MATRICES

Jun 2012

2. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$$

find **AB**.

(2)

(b) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D}$$

find the value of k for which **E** has no inverse.

(4)

9.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.

(3)

The point R has coordinates $(6, 0)$.

Given that O is the origin,

(c) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.

(4)

Jan 2013

4. The transformation U, represented by the 2×2 matrix **P**, is a rotation through $90°$ anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y=-x$.

(b) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of T as a single transformation.

(2)

6. $$\mathbf{X}=\begin{pmatrix}1 & a\\ 3 & 2\end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of a for which the matrix **X** is singular.

(2)

$$\mathbf{Y}=\begin{pmatrix}1 & -1\\ 3 & 2\end{pmatrix}$$

(b) Find $\mathbf{Y}^{-1}$.

(2)

The transformation represented by **Y** maps the point A onto the point B.

Given that B has coordinates $(1-\lambda, 7\lambda-2)$, where λ is a constant,

(c) find, in terms of λ, the coordinates of point A.

(4)

EDEXCEL FP1 - MATRICES

Jun 2013

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix $\mathbf{M}$ is singular, find the possible values of x.

(4)

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and $\mathbf{I}$ is the 2×2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by $\mathbf{A}$ maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(c) find, in terms of k, the coordinates of P.

(4)

Jun 2014

4. (i) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(a) find **AB**.

(b) Explain why $\mathbf{AB} \neq \mathbf{BA}$.

(4)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}, \text{ where } k \text{ is a real number}$$

find $\mathbf{C}^{-1}$, giving your answer in terms of k.

(3)

7. (i) In each of the following cases, find a 2×2 matrix that represents

(a) a reflection in the line $y = -x$,

(b) a rotation of 135° anticlockwise about (0, 0),

(c) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points $(1, k)$, $(3, 0)$ and $(11, 0)$, where k is a constant.

Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

Jun 2015

7. (i)

$$\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}, \text{ where } k \text{ is a real constant.}$$

Given that **A** is a singular matrix, find the possible values of k.

(4)

(ii)

$$\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$$

A triangle T is transformed onto a triangle T' by the transformation represented by the matrix **B**.

The vertices of triangle T' have coordinates $(0, 0)$, $(-20, 6)$ and $(10c, 6c)$, where c is a positive constant.

The area of triangle T' is 135 square units.

(a) Find the matrix $\mathbf{B}^{-1}$

(2)

(b) Find the coordinates of the vertices of the triangle T, in terms of c where necessary.

(3)

(c) Find the value of c.

(3)

EDEXCEL FP1 - MATRICES

Jun 2016

1. Given that k is a real number and that

$$\mathbf{A} = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix}$$

find the exact values of k for which $\mathbf{A}$ is a singular matrix. Give your answers in their simplest form.

(3)

6.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix $\mathbf{P}$.

(2)

The transformation U maps the point A, with coordinates (p, q), onto the point B, with coordinates $(6\sqrt{2}, 3\sqrt{2})$.

(b) Find the value of p and the value of q.

(3)

The transformation V, represented by the 2×2 matrix $\mathbf{Q}$, is a reflection in the line with equation $y = x$.

(c) Write down the matrix $\mathbf{Q}$.

(1)

The transformation U followed by the transformation V is the transformation T. The transformation T is represented by the matrix $\mathbf{R}$.

(d) Find the matrix $\mathbf{R}$.

(3)

(e) Deduce that the transformation T is self-inverse.

(1)