

J une 2008 6683 Statistics S1

Mark Scheme

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| Q1 <br> (a) |  | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ |
| (b) | $\begin{aligned} \mathrm{P}(\text { Positive Test }) & =0.02 \times 0.95+0.98 \times 0.03 \\ & =0.0484 \end{aligned}$ | M1A1ft <br> A1 |
| (c) | $\begin{array}{rlr} \mathrm{P}(\text { Do not have disease } \mid \text { Postive test }) & =\frac{0.98 \times 0.03}{0.0484} \\ & =0.607438 . . & \text { awrt } 0.607 \end{array}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| (d) | Test not very useful OR High probability of not having the disease for a person with a positive test | B1 <br> Total 9 |
|  | Notes: <br> (a) M1:All 6 branches. <br> Bracketed probabilities not required. <br> (b) M1 for sum of two products, at least one correct from their diagram <br> A1ft follows from the probabilities on their tree <br> A1 for correct answer only or $\frac{121}{2500}$ <br> (c) M1 for conditional probability with numerator following from their tree and denominator their answer to part (b). <br> A1 also for $\frac{147}{242}$. |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| Q5 <br> (a) | 3 closed intersecting curves with labels 100 100,30 12,10,3,25 | M1 <br> A1 <br> A1 <br> B1 <br> [4] |
|  | $\mathrm{P}($ Substance $C)=\frac{100+100+10+25}{300}=\frac{235}{300}=\frac{47}{60}$ or exact equivalent | M1A1ft |
| (c) | $\mathrm{P}($ All $3 \mid A)=\frac{10}{30+3+10+100}=\frac{10}{143}$ or exact equivalent | M1A1ft <br> [2] |
| (d) | $\mathrm{P}($ Universal donor $)=\frac{20}{300}=\frac{1}{15}$ or exact equivalent | M1A1 cao <br> [2] <br> Total 10 |
|  | Notes: <br> (a) 20 not required. Fractions and exact equivalent decimals or percentages. <br> (b) M1 For adding their positive values in $C$ and finding a probability <br> A1ft for correct answer or answer from their working <br> (c) M1 their 10 divided by their sum of values in $A$ <br> A1ft for correct answer or answer from their working <br> (d) M1 for 'their 20' divided by 300 <br> A1 correct answer only |  |


| Question Number | Scheme ${ }^{\text {a }}$ Marks |
| :---: | :---: |
| 2 |  |
| (a) (b) | M1 for $\frac{9}{25} \times \frac{2}{3}$ or $\mathrm{P}(E \mid B) \times \mathrm{P}(B)$ and at least one correct value seen. A1 for 0.24 or exact equiv. NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks. <br> $1^{\text {st }} \mathrm{M} 1$ for use of the addition rule. Must have 3 terms and some values, can ft their (a) <br> Or a full method for $\mathrm{P}\left(E^{\prime} \mid B^{\prime}\right)$ requires $1-\mathrm{P}\left(E \mid B^{\prime}\right)$ and equation for $\mathrm{P}\left(E \mid B^{\prime}\right)$ : (a) $+\frac{x}{3}=\frac{2}{5}$ <br> Or a full method for $\mathrm{P}\left(B^{\prime} \cap E\right)$ or $\mathrm{P}\left(B \cap E^{\prime}\right)$ [ or other valid method] <br> $2^{\text {nd }} \mathrm{M} 1 \quad$ for a method leading to answer e.g. $1-\mathrm{P}(E \cup B)$ $\text { or } \mathrm{P}\left(B^{\prime}\right) \times \mathrm{P}\left(E^{\prime} \mid B^{\prime}\right) \text { or } \mathrm{P}\left(B^{\prime}\right)-\mathrm{P}\left(B^{\prime} \cap E\right) \text { or } \mathrm{P}\left(E^{\prime}\right)-\mathrm{P}\left(B \cap E^{\prime}\right)$ <br> Venn Diagram $1^{\text {st }} \mathrm{M} 1$ for diagram with attempt at $\frac{2}{5}-\mathrm{P}(B \cap E)$ or $\frac{2}{3}-\mathrm{P}(B \cap E)$. Can ft their (a) <br> $1^{\text {st }}$ A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram <br> $2^{\text {nd }}$ M1 for attempting 75 - their $(18+32+12)$ <br> M1 for identifying suitable values to test for independence e.g. $\mathrm{P}(E)=0.40$ and $\mathrm{P}(E \mid B)=0.36$ <br> Or $\mathrm{P}(E) \times \mathrm{P}(B)=\ldots$ and $\mathrm{P}(E \cap B)=$ their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$ ]. Values seen somewhere <br> A1 for correct values and a correct comment <br> Diagrams You may see these or find these useful for identifying probabilities. <br> Common Errors <br> (a) $\frac{9}{25}$ is M0A0 <br> (b) $\mathrm{P}(E \cup B)=\frac{53}{75}$ scores M1A0 <br> $1-\mathrm{P}(E \cup B)=\frac{22}{75}$ scores M1A0 <br> (b) $\mathrm{P}\left(B^{\prime}\right) \times \mathrm{P}\left(E^{\prime}\right)=\frac{1}{3} \times \frac{3}{5}$ <br> scores 0/4 |
|  |  |

\begin{tabular}{|c|c|c|}
\hline Question
Number \& Scheme \& Marks \\
\hline Q2 (a) \& \multirow[t]{7}{*}{\begin{tabular}{l}
Correct tree All labels Probabilities on correct branches \\
\(\frac{1}{3} \times \frac{1}{10}=\frac{1}{30}\) or equivalent
\[
\begin{aligned}
\mathrm{CNL}+\mathrm{BNL}+\mathrm{FNL} \& =\frac{1}{2} \times \frac{4}{5}+\frac{1}{6} \times \frac{3}{5}+\frac{1}{3} \times \frac{9}{10} \\
\& =\frac{4}{5} \text { or equivalent }
\end{aligned}
\]
\[
\begin{array}{rlr}
\mathrm{P}\left(F^{\prime} / L\right) \& =\frac{\mathrm{P}\left(F^{\prime} \cap L\right)}{\mathrm{P}(L)} \quad \text { Attempt correct conditional probability but see notes } \\
\& =\frac{\frac{1}{6} \times \frac{2}{5}+\frac{1}{2} \times \frac{1}{5}}{1-(\mathrm{ii})} \& \frac{\text { numerator }}{\text { denominator }} \\
\& =\frac{\frac{5}{30}}{\frac{1}{5}}=\frac{5}{6} \& \text { or equivalent }
\end{array}
\] \\
Exact decimal equivalents required throughout if fractions not used e.g. 2(b)(i) \(0.0 \dot{3}\) Correct path through their tree given in their probabilities award Ms 2(a) All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct. \\
2(b)(i) Correct answer only award both marks. \\
2(b)(ii) At least one correct path identified and attempt at adding all three multiplied pairs award M1 \\
2(c) Require probability on numerator and division by probability for M1.Require numerator correct for their tree for M1. \\
Correct formula seen and used, accept denominator as attempt and award M1 No formula, denominator must be correct for their tree or 1-(ii) for M1 \(1 / 30\) on numerator only is M0, \(\mathrm{P}\left(\mathrm{L} / \mathrm{F}^{\prime}\right)\) is M0.
\end{tabular}} \& B1
B1
B1

(3) \\
\hline (b)(i) \& \& M1 A1
M1 \\
\hline \& \& A1 (2) \\
\hline (c) \& \& M1 \\
\hline \& \& $\frac{\mathrm{A} 1}{\mathrm{~A} 1 \mathrm{ft}}$ \\

\hline \& \& | A1 |
| :--- | ---: |
|  |
|  |
|  | \\

\hline Notes \& \& \\
\hline
\end{tabular}

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| Q7(a) (i) ${ }^{\text {(ii) }}$ (b) | $\mathrm{P}(A \cup B)=a+b$ cao | B1 |
|  | $\mathrm{P}(A \cup B)=a+b-a b \quad$ or equivalent | B1 (2) |
|  | $\begin{aligned} \mathrm{P}(R \cup Q) & =0.15+0.35 \\ & =0.5 \end{aligned}$ | B1 |
|  | $\begin{gathered} \mathrm{P}(R \cap Q)=\mathrm{P}(R \mid Q) \times \mathrm{P}(Q) \\ =0.1 \times 0.35 \end{gathered}$ | M1 |
|  | $=0.035$ 0.035 | A1 |
|  | $\mathrm{P}(R \cup Q)=\mathrm{P}(R)+\mathrm{P}(Q)-\mathrm{P}(R \cap Q) \quad$ OR $\quad \mathrm{P}(R)=\mathrm{P}\left(R \cap Q^{\prime}\right)+\mathrm{P}(R \cap Q)$ | M1 |
|  | $\begin{array}{lll} \mathrm{P}(R)=0.185 & =0.185 & 0.185 \end{array}$ | A1 |
|  |  | $\begin{aligned} & (2) \\ & {[7]} \end{aligned}$ |
| Notes |  |  |
|  | 7(a) (i) Accept $a+b-0$ for B1 <br> Special Case <br> If answers to (i) and (ii) are <br> (i) $\mathrm{P}(A)+\mathrm{P}(B)$ and (ii) $\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A) \mathrm{P}(B)$ <br> award B0B1 <br> 7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded. |  |


| Question Number | Scheme Marks |
| :---: | :---: |
| Q1 (a) |  |
| (a) <br> (b) <br> Special Case | M1 for shape and labels: 3 branches followed by $3,2,2$ with some $R, B$ and $G$ seen Allow 3 branches followed by $3,3,3$ if 0 probabilities are seen implying that $3,2,2$ intended Allow blank branches if the other probabilities imply probability on blanks is zero <br> Ignore further sets of branches <br> $1^{\text {st }}$ A1 for correct probabilities and correct labels on $1^{\text {st }}$ set of branches. <br> $2^{\text {nd }} \mathrm{A} 1$ for correct probabilities and correct labels on $2^{\text {nd }}$ set of branches. (accept $0.33,0.67$ etc or better here) <br> M1 for identifying the 2 cases $B G$ and $G B$ and adding 2 products of probabilities. These cases may be identified by their probabilities e.g. $\left(\frac{1}{4} \times \frac{1}{3}\right)+\left(\frac{1}{4} \times \frac{1}{3}\right)$ NB $\frac{1}{6}$ (or exact equivalent) with no working scores $2 / 2$ <br> With Replacement (This oversimplifies so do not apply Mis-Read: max mark 2/5) <br> (a) B1 for 3 branches followed by 3, 3, 3 with correct labels and probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ on each. <br> (b) M1 for identifying 2, possibly correct cases and adding 2 products of probabilities but A0 for wrong answer <br> $\left[\left(\frac{1}{4} \times \frac{1}{4}\right)+\left(\frac{1}{4} \times \frac{1}{4}\right)\right]$ will be sufficient for M1A0 here but $\frac{1}{4} \times \frac{1}{2}+\ldots$ would score M0 |



| Question <br> Number | Scheme ${ }^{\text {Scorks }}$ |
| :---: | :---: |
| (a) <br> (b) <br> (c) <br> (d) | $\mathrm{P}(R)$ and $\mathrm{P}(B)$ <br> $\mathrm{P}(H)=\frac{5}{12} \times \frac{2}{3}+\frac{7}{12} \times \frac{1}{2},=\frac{41}{72}$ or awrt 0.569 <br> $\mathrm{P}(R \mid H)=\frac{\frac{5}{12} \times \frac{2}{3}}{" \frac{41}{72} "},=\frac{20}{41}$ or awrt 0.488 <br> $\left(\frac{5}{12}\right)^{2}+\left(\frac{7}{12}\right)^{2}$ <br> $=\frac{25}{144}+\frac{49}{144}=\frac{74}{144}$ or $\frac{37}{72}$ or awrt 0.514 |
| (a) <br> (b) <br> (c) <br> Formula seen <br> Formula not seen <br> (d) | $1^{\text {st }} \mathrm{B} 1 \quad$ for the probabilities on the first 2 branches. Accept $0.41 \dot{6}$ and $0.58 \dot{3}$ <br> $2^{\text {nd }}$ B1 for probabilities on the second set of branches. Accept $0 . \dot{6}, 0 . \dot{3}, 0.5$ and $\frac{1.5}{3}$ <br> Allow exact decimal equivalents using clear recurring notation if required. <br> M1 for an expression for $\mathrm{P}(H)$ that follows through their sum of two products of probabilities from their tree diagram <br> M1 for $\frac{\mathrm{P}(R \cap H)}{\mathrm{P}(H)}$ with denominator their (b) substituted e.g. $\frac{\mathrm{P}(R \cap H)}{\mathrm{P}(H)}=\frac{\frac{5}{12}}{\text { (their (b)) }}$ award M1. <br> M1 for $\frac{\text { probability } \times \text { probability }}{\text { their } b}$ but M0 if fraction repeated e.g. $\frac{\frac{5}{12} \times \frac{2}{3}}{\frac{2}{3}}$. <br> $1^{\text {st }}$ A1ft for a fully correct expression or correct follow through <br> $2^{\text {nd }}$ A1 for $\frac{20}{41}$ o.e. <br> M1 for $\left(\frac{5}{12}\right)^{2}$ or $\left(\frac{7}{12}\right)^{2}$ can follow through their equivalent values from tree diagram <br> $1^{\text {st }}$ A1 for both values correct or follow through from their original tree and + <br> $2^{\text {nd }}$ A1 for a correct answer <br> Special Case $\frac{5}{12} \times \frac{4}{11}$ or $\frac{7}{12} \times \frac{6}{11}$ seen award M1A0A0 |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| Q4 (a) | $\frac{2+3}{\text { their total }}=\frac{5}{\text { their total }}=\frac{1}{6}\left(* * \text { given answer }{ }^{* *}\right)$ | M1 A1cso |
| (b) | $\frac{4+2+5+3}{\text { total }},=\frac{14}{30} \text { or } \frac{7}{15} \text { or } 0.4 \dot{6}$ | M1 A1 <br> (2) |
| (c) | $\mathrm{P}(A \cap C)=0$ | B1 <br> (1) |
| (d) | $\mathrm{P}(\mathrm{C} \mid \text { reads at least one magazine })=\frac{6+3}{20}=\frac{9}{20}$ | M1 A1 |
| (e) | $\mathrm{P}(B)=\frac{10}{30}=\frac{1}{3}, \mathrm{P}(C)=\frac{9}{30}=\frac{3}{10}, \quad \mathrm{P}(B \cap C)=\frac{3}{30}=\frac{1}{10} \quad \text { or } \mathrm{P}(B \mid C)=\frac{3}{9}$ | M1 |
|  | $\mathrm{P}(B) \times \mathrm{P}(C)=\frac{1}{3} \times \frac{3}{10}=\frac{1}{10}=\mathrm{P}(B \cap C) \quad \text { or } \mathrm{P}(B \mid C)=\frac{3}{9}=\frac{1}{3}=\mathrm{P}(B)$ | M1 |
|  | So yes they are statistically independent | Alcso |
|  |  | Total 10 |
| (a) | M1 for $\frac{2+3}{\text { their total }}$ or $\frac{5}{30}$ |  |
| (b) | M1 for adding at least 3 of " $4,2,5,3$ " and dividing by their total to give a probability Can be written as separate fractions substituted into the completely correct Addition Rule |  |
| (c) | B1 for 0 or $0 / 30$ |  |
| (d) | M1 for a denominator of $\mathbf{2 0}$ or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, $2 / 2$ |  |
| (e) | $1^{\text {st }} \mathrm{M} 1$ for attempting all the required probabilities for a suitable test $2^{\text {nd }} \mathrm{M} 1$ for use of a correct test - must have attempted all the correct probabilities. Equality can be implied in line 2. <br> A1 for fully correct test carried out with a comment |  |

