Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt}=1600-c\sqrt{h}$ or $\frac{dV}{dt}=1600-k\sqrt{h}$,	Either of these statements	M1
	$(V=4000h \ \Rightarrow)\ \frac{dV}{dh}=4000$	$\frac{dV}{dh}=4000$ or $\frac{dh}{dV}=\frac{1}{4000}$	M1
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt}=\frac{1600-c\sqrt{h}}{4000}=\frac{1600}{4000}-\frac{c\sqrt{h}}{4000}=0.4-k\sqrt{h}$ or $\frac{dh}{dt}=\frac{1600-k\sqrt{h}}{4000}=\frac{1600}{4000}-\frac{k\sqrt{h}}{4000}=0.4-k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h=25$ water ***leaks out such that*** $\frac{dV}{dt}=400$		
	$400=c\sqrt{h} \Rightarrow 400=c\sqrt{25} \Rightarrow 400=c(5) \Rightarrow c=80$		
	From above; $k=\frac{c}{4000}=\frac{80}{4000}=0.02$ as required	Proof that $k=0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400=4000k\sqrt{h}$		
	$\Rightarrow 400=4000k\sqrt{25}$		
	$\Rightarrow 400=k(20000) \Rightarrow k=\frac{400}{20000}=0.02$	Using 400, 4000 and $h=25$ or $\sqrt{h}=5$. Proof that $k=0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt}=0.4-k\sqrt{h} \Rightarrow \int\frac{dh}{0.4-k\sqrt{h}}=\int dt$	***Separates the variables*** with $\int\frac{dh}{0.4-k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $=\int_0^{100}\frac{1}{0.4-0.02\sqrt{h}}\,dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $=\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh$	Correct proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$		
	$\frac{\mathrm{d}h}{\mathrm{d}x} = 2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x} = -2(20-x)$	Correct $\frac{\mathrm{d}h}{\mathrm{d}x}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$	$\pm\lambda\int\frac{20-x}{x}\,\mathrm{d}x$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,\mathrm{d}x$ where λ is a constant	M1
	$=100\int\frac{x-20}{x}\,\mathrm{d}x$		
	$=100\int\left(1-\frac{20}{x}\right)\mathrm{d}x$		
	$=100(x-20\ln x)\ \ (+c)$	$\pm\alpha x\pm\beta\ln x\ ;\ \alpha,\beta\neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100x-2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$=\left(1000-2000\ln 10\right)-\left(2000-2000\ln 20\right)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$=2000\ln 20-2000\ln 10-1000$	Combining logs to give...	
	$=2000\ln 2-1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 **aef**
			[6]
(e)	Time required $=2000\ln 2-1000 = 386.2943611...$ sec		
	= 386 seconds (nearest second)		
	= 6 minutes and 26 seconds (nearest second)	6 minutes, 26 seconds	B1
			[1]
			13 marks

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	$\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$, aef	A1 **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)		**[3]**

Question Number	Scheme		Marks
7. (b)	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$	their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their *A* and *B* and no error with the "2" with or without $+\,c$	A1 $\sqrt{}$
	$y = 0,\ x = \frac{\pi}{3} \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y = 0$ and $x = \frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

Question Number	Scheme		Marks	
Q5	(a) $\int \frac{9x+6}{x}\,dx = \int \left(9 + \frac{6}{x}\right) dx$		M1	
	$= 9x + 6\ln x \quad (+C)$		A1	(2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,dy = \int \frac{9x+6}{x}\,dx$	Integral signs not necessary	B1	
	$\int y^{-\frac{1}{3}}\,dy = \int \frac{9x+6}{x}\,dx$			
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \quad (+C)$	$\pm k y^{\frac{2}{3}} =$ their (a)	M1	
	$\frac{3}{2} y^{\frac{2}{3}} = 9x + 6\ln x \quad (+C)$	ft their (a)	A1ft	
	$y = 8,\ x = 1$			
	$\frac{3}{2} 8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

Question Number	Scheme		Marks	
8.	(a) $\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1	
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$		B1	
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1	
	Leading to $75\dfrac{dh}{dt} = 4 - 5h$ ∗	cso	A1	**(5)**
	(b) $\int \dfrac{75}{4-5h}\,dh = \int 1\,dt$	separating variables	M1	
	$-15\ln(4-5h) = t \quad (+C)$		M1 A1	
	$-15\ln(4-5h) = t + C$			
	When $t = 0$, $h = 0.2$			
	$-15\ln 3 = C$		M1	
	$t = 15\ln 3 - 15\ln(4-5h)$			
	When $h = 0.5$			
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1	
	Alternative for last 3 marks			
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$			
	$= -15\ln 1.5 + 15\ln 3$		M1 M1	
	$= 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1	**(6)**