7) a) $P = \dfrac{2}{2-\sin t}$ $\quad t=0 \Rightarrow P=1$

i) Max $P = \dfrac{2}{2-1} = 2$

Min $P = \dfrac{2}{2-(-1)} = \dfrac{2}{3}$

ii)

$$P = \frac{2}{2-\sin t}$$

$$\frac{dP}{dt} = \frac{(2-\sin t)0 - 2(-\cos t)}{(2-\sin t)^2}$$

$$\frac{dP}{dt} = \frac{2\cos t}{(2-\sin t)^2}$$

$$\frac{dP}{dt} = \frac{1}{2} \times \frac{4}{(2-\sin t)^2}\cos t$$

$$\frac{dP}{dt} = \frac{1}{2}P^2\cos t$$

$\therefore$ P satisfies this diff. eqn.

7b)

$$\frac{dP}{dt} = \frac{1}{2}\left(2P^2 - P\right)\cos t$$

$P=1$ when $t=0$

i) Let $\dfrac{1}{P(2P-1)} \equiv \dfrac{A}{P} + \dfrac{B}{2P-1}$

$\Rightarrow 1 \equiv A(2P-1) + BP$

When $P=0$

$1 = -A \quad \Rightarrow A = -1$

When $P = \frac{1}{2}$

$1 = \frac{1}{2}B \quad \Rightarrow B = 2$

$$\frac{1}{P(2P-1)} \equiv \frac{2}{2P-1} - \frac{1}{P}$$

ii)

$$\int \frac{1}{2P^2-P}\,dP = \int \frac{1}{2}\cos t\,dt$$

$$\int\left(\frac{2}{2P-1} - \frac{1}{P}\right)dP = \int \frac{1}{2}\cos t\,dt$$

$$\ln(2P-1) - \ln P = \frac{1}{2}\sin t + c$$

$$\ln\left(\frac{2P-1}{P}\right) = \frac{1}{2}\sin t + c$$

$P=1$, when $t=0$ so

$$\ln\left(\frac{1}{1}\right) = \frac{1}{2}\sin 0 + c$$

$$0 = 0 + c \Rightarrow c = 0$$

$$\therefore \ln\left(\frac{2P-1}{P}\right) = \frac{1}{2}\sin t$$

iii)

$$P = \frac{1}{2 - e^{\frac{1}{2}\sin t}}$$

Min $P = \dfrac{1}{2-e^{-0.5}}$

$= 0.718$ to 3 d.p

Max $P = \dfrac{1}{2-e^{0.5}}$

$= 2.847$ to 3 d.p

8) $x = 10\cos\theta + 5\cos 2\theta$

$y = 10\sin\theta + 5\sin 2\theta$

$(0 \le \theta < 2\pi)$

i) $\frac{dy}{d\theta} = 10\cos\theta + 10\cos 2\theta$

$\frac{dx}{d\theta} = -10\sin\theta - 10\sin 2\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{10\cos\theta + 10\cos 2\theta}{-10\sin\theta - 10\sin 2\theta}$

$\frac{dy}{dx} = -\frac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$

When $\theta = \frac{\pi}{3}$

$\frac{dy}{dx} = -\frac{\cos\frac{\pi}{3} + \cos\frac{2\pi}{3}}{\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}}$

$= -\left(\frac{\frac{1}{2} + -\frac{1}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}\right)$

$= 0$

At A, $\frac{dy}{dx} = 0$

$x = 10\cos\frac{\pi}{3} + 5\cos\frac{2\pi}{3}$

$x = 10 \times \frac{1}{2} + 5(-\frac{1}{2}) = 5/2$

$y = 10\sin\frac{\pi}{3} + 5\sin\frac{2\pi}{3}$

$y = 10 \times \frac{\sqrt{3}}{2} + 5 \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$

A is point $\left(\frac{5}{2}, \frac{15\sqrt{3}}{2}\right)$

8ii) $x^2 + y^2 =$

$\left(10\cos\theta + 5\cos 2\theta\right)^2 + \left(10\sin\theta + 5\sin 2\theta\right)^2$

$= 100\cos^2\theta + 100\cos\theta\cos 2\theta + 25\cos^2 2\theta$

$+ 100\sin^2\theta + 100\sin\theta\sin 2\theta + 25\sin^2 2\theta$

$= 100\left(\cos^2\theta + \sin^2\theta\right) + 25\left(\cos^2 2\theta + \sin^2 2\theta\right)$

$+ 100\left(\cos\theta\cos 2\theta + \sin\theta\sin 2\theta\right)$

$= 100 + 25 + 100\cos(2\theta - \theta)$

$= 125 + 100\cos\theta$

8iii) (Distance from O)$^2 = x^2 + y^2$

Max Dist^2 = 125 + 100 = 225

Max Distance = 15 m

Min Dist^2 = 125 − 100 = 25

Min Distance = 5 m

8iv) $2\cos^2\theta + 2\cos\theta - 1 = 0$

By calculator $\cos\theta = 0.3660254$

$\cos\theta = -1.3660254$

Now $x^2 + y^2 = 125 + 100\cos\theta$

and at B, $x = 0$

$\therefore y^2 = 125 + 100 \times 0.3660254$

$\Rightarrow y^2 = 161.60254$

$\Rightarrow y = 12.712$

$\therefore$ OB = 12.7 m to 3 s.f.