Section B

7(i) At A, $\cos\theta = 1 \Rightarrow \theta = 0$ At B, $\cos\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow \quad y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ $dy/dx = 0$ when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow \quad 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow \quad 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ ($1 + ½\sqrt{6} > 1$ so no solution) $\Rightarrow \quad \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ $= 1.35\pi = 4.24$	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ $= 60$ m	M1 A1 [2]	
(ii) $\overrightarrow{BA} = \begin{pmatrix}-40\\-40\\20\end{pmatrix} = 20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}} = -\dfrac{13}{3\sqrt{26}}$ $\Rightarrow \quad \theta = 148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ −13 oe eg -260 √9√26 oe eg 60√26 cao (or radians)
(iii) $\mathbf{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z = 0 \Rightarrow \lambda = 20$ $\Rightarrow \quad a = 40 + 3 \times 20 = 100$ $b = 0 + 4 \times 20 = 80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….+$\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix} = -12+10+2 = 0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2 = 0$ $\Rightarrow \begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x - 5y + 2z = c$ At B (say) $6 \times 40 - 5 \times 0 + 2 \times -20 = c$ $\Rightarrow c = 200$ so $6x - 5y + 2z = 200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)