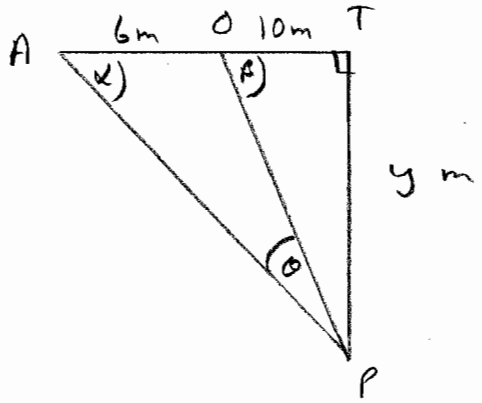
Section B

7 i)



In Δ OTP

$\angle TPO = 90 - \beta$

In Δ ATP

$\alpha + 90 + 90 - \beta + \theta = 180°$

$\Rightarrow \alpha - \beta + \theta = 0$

$\Rightarrow \theta = \beta - \alpha$

$$\tan\theta = \tan(\beta - \alpha)$$

$$= \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$$

But $\tan\beta = \frac{y}{10}$ and $\tan\alpha = \frac{y}{16}$

$$\therefore \tan\theta = \frac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10} \times \frac{y}{16}}$$

$$\tan\theta = \frac{\frac{16y}{160} - \frac{10y}{160}}{1 + \frac{y^2}{160}}$$

$$\tan\theta = \frac{\frac{6y}{160}}{\frac{160 + y^2}{160}}$$

$$= \frac{6y}{160 + y^2}$$

When $y = 6$

$$\tan\theta = \frac{36}{196}$$

$$\theta = 10.4°$$

6 ii) $\tan\theta = \dfrac{6y}{160+y^2}$

$$\sec^2\theta \frac{d\theta}{dy} = \frac{(160+y^2)6 - 6y(2y)}{(160+y^2)^2}$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dy} = \frac{6(160+y^2)-12y^2}{(160+y^2)^2}$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dy} = \frac{6(160+y^2-2y^2)}{(160+y^2)^2}$$

$$\frac{d\theta}{dy} = \frac{6(160-y^2)}{(160+y^2)^2}\cos^2\theta$$

6 iii) θ a max when $\dfrac{d\theta}{dy} = 0$

$\dfrac{d\theta}{dy} = 0$ when $160 - y^2 = 0$

$\Rightarrow y^2 = 160$

$\Rightarrow y = \sqrt{160}$

$y \approx 12.65$ m

to 4 sig fig.

8)

$$x = \frac{a}{1+kt}$$

Given when $t=0$, $x = 2.5$

i) $\dfrac{dx}{dt} = \dfrac{(1+kt)0 - ak}{(1+kt)^2}$

$$\frac{dx}{dt} = \frac{-ak}{(1+kt)^2}$$

$$= \frac{-ka^2}{a(1+kt)^2} = \frac{-kx^2}{a}$$

ii) when $t=0$, $x = 2.5$

$\Rightarrow 2.5 = \dfrac{a}{1+0}$

$\Rightarrow a = 2.5$

when $t=1$, $x = 1.6$

$\Rightarrow 1.6 = \dfrac{2.5}{1+k}$

$(1+k)1.6 = 2.5$

$1+k = \dfrac{2.5}{1.6}$

$k = \dfrac{2.5}{1.6} - 1$

$k = 0.5625$

iii)

$$x = \frac{2.5}{1+0.5625t}$$

As $t \to \infty$, $x \to 0$

In the long term this predicts red squirrels die out

8 iv) $\frac{1}{2y-y^2} = \frac{1}{y(2-y)}$

$\frac{1}{y(2-y)} \equiv \frac{A}{y} + \frac{B}{2-y}$

$1 \equiv A(2-y) + By$

when $y=2$

$1 = 2B \Rightarrow B = \frac{1}{2}$

when $y=0$

$1 = 2A \Rightarrow A = \frac{1}{2}$

$\therefore \frac{1}{2y-y^2} \equiv \frac{1}{2y} + \frac{1}{2(2-y)}$

v) $\frac{dy}{dt} = 2y - y^2$

$\int \frac{1}{2y-y^2}\,dy = \int 1\,dt$

$\int \left(\frac{1}{2y} + \frac{1}{2(2-y)}\right) dy = \int 1\,dt$

$\frac{1}{2}\ln y - \frac{1}{2}\ln(2-y) = t + c$

Given $t=0$, $y=1$

$\frac{1}{2}\ln 1 - \frac{1}{2}\ln 1 = 0 + c$

$\Rightarrow c = 0$

$\therefore \frac{1}{2}\ln y - \frac{1}{2}\ln(2-y) = t$

$\frac{1}{2}\ln\left(\frac{y}{2-y}\right) = t$

$\ln\left(\frac{y}{2-y}\right) = 2t$

$\frac{y}{2-y} = e^{2t}$

$y = e^{2t}(2-y)$

$y = 2e^{2t} - ye^{2t}$

$y + ye^{2t} = 2e^{2t}$

$y(1+e^{2t}) = 2e^{2t}$

$y = \frac{2e^{2t}}{(1+e^{2t})}$

$\Rightarrow y = \frac{2}{\frac{1}{e^{2t}} + 1}$

$\Rightarrow y = \frac{2}{1+e^{-2t}}$

vi) As $t \to \infty$, $y \to 2$

Long term population of grey squirrels $\to 2000$