Section B

7(i) $A\hat{O}P = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \quad \beta = \alpha + \theta$ $\Rightarrow \quad \theta = \beta - \alpha$ $\tan\theta = \tan(\beta - \alpha)$ $= \dfrac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ $= \dfrac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}.\frac{y}{16}}$ $= \dfrac{16y - 10y}{160 + y^2}$ $= \dfrac{6y}{160 + y^2}$ * When $y = 6$, $\tan\theta = 36/196$ $\Rightarrow \quad \theta = 10.4°$	M1 M1 E1 M1 A1 E1 M1 A1 cao [8]	Use of sum of angles in triangle OPT and AOP oe SC B1 for $\beta=\alpha+\theta$, $\theta=\beta-\alpha$ no justification Use of Compound angle formula Substituting values for tan α and tan β www accept radians
(ii) $\sec^2\theta \dfrac{d\theta}{dy} = \dfrac{(160 + y^2)6 - 6y.2y}{(160 + y^2)^2}$ $= \dfrac{6(160 + y^2 - 2y^2)}{(160 + y^2)^2}$ $\Rightarrow \quad \dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$ *	M1 M1 A1 A1 E1 [5]	$\sec^2\theta \dfrac{d\theta}{dy} = \ldots$ quotient rule correct expression simplifying numerator www
(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow \quad y^2 = 160$ $\Rightarrow \quad y = 12.65$ When $y = 12.65$, $\tan\theta = 0.237\ldots$ $\Rightarrow \quad \theta = 13.3°$	M1 A1 M1 A1cao [4]	 oe accept radians

8 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt = a/x - 1$, $t = a/kx - 1/k$ $dt/dx = -a/kx^2$ $\Rightarrow dx/dt = -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y - y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y - y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y - y^2} dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}] dy = \int dt$ $\Rightarrow$ $½ \ln y - ½ \ln(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables $½ \ln y - ½ \ln(2 - y)$ ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$