1. (a) $2\log_3(x-5) = \log_3(x-5)^2$ B1

$\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$ M1

$\log_3 3 = 1$ seen or used correctly B1

$\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$ $\left\{\frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13)\right\}$ M1

$x^2 - 16x + 64 = 0$ (*) A1 cso 5

Note

Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3 (x-5)^2 - \log_3 (2x-13) = \frac{\log_3 (x-5)^2}{\log_3 (2x-13)}$ is M0

$2\log_3 (x-5) - \log_3 (2x-13) = 2\log \frac{x-5}{2x-13}$ is M0

2nd M: After the first mistake above, this mark is available only if there is 'recovery' to the required

$\log_3 \left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q$. Even then the final mark (cso) is lost.

'Cancelling logs', e.g. $\frac{\log_3 (x-5)^2}{\log_3 (2x-13)} = \frac{(x-5)^2}{2x-13}$ will also lose the 2nd M.

A typical wrong solution:

$\log_3 \frac{(x-5)^2}{2x-13} = 1 \Rightarrow \log_3 \frac{(x-5)^2}{2x-13} = 3(*) \Rightarrow \frac{(x-5)^2}{2x-13} = 3$

$\Rightarrow (x-5)^2 = 3(2x-13)$

(*) Wrong step here

This, with no evidence elsewhere of $\log_3 3 = 1$, scores B1 M1 B0 M0 A0

However, $\log_3 \frac{(x-5)^2}{2x-13} = 1 \Rightarrow \frac{(x-5)^2}{2x-13} = 3$ is correct and could lead to full marks.

(Here $\log_3 3 = 1$ is implied).

No log methods shown:

It is not acceptable to jump immediately to $\frac{(x-5)^2}{2x-13} = 3$. The only mark this scores is the 1st B1 (by generous implication).

(b) $(x-8)(x-8)=0 \quad \Rightarrow \quad x=8$ Must be seen in part (b). M1 A1

Or: Substitute $x = 8$ into original equation and verify.
Having additional solution(s) such as $x = -8$ loses the A mark. 2
$x = 8$ with no working scores both marks.

Note

M1: Attempt to solve the given quadratic equation (usual rules), so the factors $(x - 8)(x - 8)$ with no solution is M0.

[7]

2. (a) $\log_x 64 = 2 \Rightarrow 64 = x^2$ M1

So $x = 8$ A1 2

Note

M1 for getting out of logs

A1 Do not need to see $x = -8$ appear and get rejected. Ignore $x = -8$ as extra solution. $x= 8$ with no working is M1 A1

Alternatives

Change base : (i) $\dfrac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or

(ii) $\dfrac{\log_{10} 64}{\log_{10} x} = 2, \log x = \frac{1}{2} \log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1

BUT $\log x = 0.903$ so $x = 8$ is M1A0 (loses accuracy mark)

(iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1

(b) $\log_2(11-6x) = \log_2(x-1)^2 + 3$ M1

$\log_2\left[\dfrac{11-6x}{(x-1)^2}\right] = 3$ M1

$\dfrac{11-6x}{(x-1)^2} = 2^3$ M1

$\{11 - 6x = 8(x^2 - 2x + 1)\}$ and so $0 = 8x^2 - 10x - 3$ A1

$0 = (4x+1)(2x-3) \Rightarrow x = ...$ dM1

$x = \dfrac{3}{2}, \left[-\dfrac{1}{4}\right]$ A1 6

Note

1st M1 for using the $n\log x$ rule

2nd M1 for using the $\log x - \log y$ rule or the $\log x + \log y$ rule as appropriate

3rd M1 for using 2 to the power– need to see 2^3 or 8 (May see $3 = \log_2 8$ used)

If all three M marks have been earned and logs are still present in equation **do not give** final M1. So solution stopping at

$\log_2\left[\frac{11-6x}{(x-1)^2}\right] = \log_2 8$ would earn M1M1M0

1st A1 for a correct 3TQ

4th dependent M1 for attempt to solve or factorize their 3TQ to obtain $x =$… (mark depends on three previous M marks)

2nd A1 for 1.5 (ignore –0.25)

s.c 1.5 only – no working – is 0 marks

[8]

3. $2\log_5 x = \log_5 (x^2)$, $\log_5 (4-x) - \log_5 (x^2) = \log_5 \frac{4-x}{x^2}$ B1 M1

$\log\left(\frac{4-x}{x^2}\right) = \log 5$ $5x^2 + x - 4 = 0$ **or** $5x^2 + x = 4$ **o.e.** M1 A1

$(5x-4)(x+1) = 0$ $x = \frac{4}{5}$ $(x = -1)$ dM1 A1 6

Alternative 1

$\log_5 (4-x) - 1 = 2\log_5 x$ so $\log_5 (4-x) - \log_5 5 = 2\log_5 x$ M1

$\log_5 \frac{4-x}{5} = 2\log_5 x$ M1

then could complete solution with $2\log_5 x = \log_5(x^2)$ B1

$\left(\frac{4-x}{5}\right) = x^2$ $5x^2 + x - 4 = 0$ A1

Then as in first method $(5x-4)(x+1) = 0$ $x = \frac{4}{5}$ $(x = -1)$ dM1 A1 6

Notes

B1 is awarded for 2log x = log x^2 anywhere.

M1 for correct use of log A – log B = log $\frac{A}{B}$

M1 for replacing 1 by $\log_k k$. **A1** for correct quadratic

(log(4 – x) – logx^2 = log5 $\Rightarrow$ 4 – x – x^2 = 5 is **B1M0M1A0 M0A0)**

dM1 for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded)

A1 for 4/5 or 0.8 or equivalent (Ignore extra answer).

Special cases

Complete trial and error yielding 0.8 is **M3** and **B1** for 0.8 **A1, A1** awarded for each of two tries evaluated. i.e. 6/6
Incomplete trial and error with wrong or no solution is 0/6
Just answer 0.8 with no working is **B1**
If log base 10 or base e used throughout – can score **B1M1M1A0M1A0**

[6]

4. Method 1 (Substituting a = 3b into second equation at some stage)

Using a law of logs correctly (anywhere)	e.g. $\log_3 ab = 2$	M1
Substitution of 3b for a (or a/3 for b)	e.g. $\log_3 3b^2 = 2$	M1
Using base correctly on **correctly derived** $\log_3$ p = q	e.g. $3b^2 = 3^2$	M1
First correct value	$b = \sqrt{3}$ (allow $3^{½}$)	A1
Correct method to find other value (dep. on at least first M mark)		
Second answer	$a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1

Method 2 (Working with two equations in $\log_3$a and $\log_3$b)

"Taking logs" of first equation and "separating" $\log_3 a = \log_3 3 + \log_3 b$ (= 1 + $\log_3 b$)	M1		
Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a$ = 1½, $\log_3 b$ = ½]		M1	
Using base correctly to find a or b	M1		
Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1		
Correct method for second answer, dep. on first M; correct second answer	M1; A1		6
[Ignore negative values]			

Answers must be exact; decimal answers lose both A marks
There are several variations on **Method 1**, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme.
In this method, the first three method marks on Epen are for

(i) First M1: correct use of log law,

(ii) Second M1: substitution of $a = 3b$,

(iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$

Three examples of applying first 4 marks in Method 1:

(i) $\log_3 3b + \log_3 b = 2$ gains second M1
$\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1
($2 \log_3 b = 1$, $\log_3 b = ½$) no mark yet
$b = 3^{½}$ gains third M1, and if correct A1

(ii) $\log_3 (ab) = 2$ gains first M1
$ab = 3^2$ gains third M1
$3b^2 = 3^2$ gains second M1

(iii) $\log_3 3b^2 = 2$ has gained first 2 M marks
$\Rightarrow 2 \log_3 3b = 2$ or similar type of error
$\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly

[6]

5.	(i)	2	B1	1
	(ii)	$2\log3 = \log3^2$ (or $2\log p = \log p^2$)	B1	
		$\log_a p + = \log_a 11 = \log_a 11p = \log_a 99$ (Allow e.g. $\log_a(3^2 \times 11)$)	M1,A1	3

Ignore 'missing base' or wrong base.
The correct answer with no working scores full marks
$\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.

[4]

6.	(a)	$\log 5^x = \log 8$ or $x = \text{lo}_5\ 8$	M1	
		Complete method for finding x: $x = \dfrac{\log 8}{\log 5}$ or $\dfrac{\ln 8}{\ln 5}$	M1	
		= 1.29 only	A1	3

	(b)	Combining two logs: $\log_2 \frac{(x+1)}{x}$ or $\log_2 7x$	M1	
		Forming equation in x (eliminating logs) legitimately	M1	
		$x = \frac{1}{6}$ or $0.1\dot{6}$	A1	3

[6]

7.	(a)	$\log 3^x = \log 5$	M1	
		$x = \frac{\log 5}{\log 3}$ or $x \log 3 = \log 5$	A1	
		$= \underline{1.}46$	A1 cao	3
	(b)	$\log_2 (\frac{2x+1}{x}) = 2$	M1	
		$\frac{2x+1}{x} = 2^2$ or 4	M1	
		$2x + 1 = 4x$	M1	
		$x = \frac{1}{2}$ or 0.5	A1	4

[7]

8.	(a)	$\log_5 x^2 - \log_5 y$; $= 2\log_5 x - \log_5 y = \mathbf{2a - b}$	M1A1	2
	(b)	$\log_5 25 = 2$ or $\log_5 y$	B1	
		$\log_5 25 + \log_5 x + \log_5 y^{\frac{1}{2}}$; $= \mathbf{2 + a + ½ b}$	M1;A1	3
	(c)	$2a - b = \mathbf{1}$, $2 + a + ½ b = \mathbf{1}$ (must be in a and b)	B1 ft	1
	(d)	Using both **correct** equations to show that $a = -0.25$ (*) $b = -1.5$ *[Mark for (c) can be gained in (d)]*	M1 B1	 2
	(e)	Using correct method to find a value for x or a value of y: $x = 5^{-0.25} = \mathbf{0.669}$, $y = 5^{-1.5} = \mathbf{0.089}$ *[max. penalty –1 for more than 3 d.p.]*	M1 A1 A1 ft	 3

[11]

9. (a) $\log_2 (16x) = \log_2 16 + \log_2 x$ M1

$= \underline{4 + a}$ A1 c.a.o 2

M1 Correct use of $\log(ab) = \log_a + \log_b$

(b) $\log_2 \left(\frac{x^4}{2}\right) = \log_2 x^4 - \log_2 2$ M1

$= 4 \log 2x - \log_2 2$ M1

$= \underline{4a - 1}$ (accept $4 \log_2 x - 1$) M1 3

M1 Correct use of $\log\left(\frac{a}{b}\right) = \ldots$

M1 Use of $\log x^n = n \log x$

(c) $\frac{1}{2} = 4 + a - (4a - 1)$ M1

$a = \frac{3}{2}$ A1

$\log_2 x = \frac{3}{2} \quad \Rightarrow \quad x = 2^{\frac{3}{2}}$ M1

$\underline{x = \sqrt{8} \text{ or } \sqrt{2^3} \text{ or } (\sqrt{2})^3}$ A1 4

M1 Use their (a) & (b) to form equ in a

M1 Out of logs: $x = 2^a$

A1 Must write x in surd form, follow through their rational a.

[9]

10. (a) $\frac{x^2 + 4x + 3}{x^2 + x} = \frac{(x+3)(x+1)}{x(x+1)}$ M1

Attempt to factorise numerator or denominator

$= \underline{\frac{x+3}{x}}$ or $1 + \frac{3}{x}$ or $(x + 3)x^{-1}$ A1 2

(b) LHS = $\log_2\left(\frac{x^2+4x+3}{x^2+x}\right)$ M1 (*)

Use of log a – log b

RHS = 2^4 or 16 B1

$x + 3 = 16x$ M1 (*)

Linear or quadratic equation in x

(*) *dep*

$x = \frac{3}{15}$ or $\frac{1}{5}$ or 0.2 A1 4

[6]

11. $\log_3 x^2 - \log_3 (x - 2) = 2$

Use of log x^n rule M1

$\log_3\left(\frac{x^2}{x-2}\right) = 2$

Use of log a – log b rule M1

$\frac{x^2}{x-2} = 3^2$

Getting out of logs M1

$x^2 - 9x + 18 = 0$

Correct 3TQ = 0 A1

$(x - 6)(x - 3) = 0$

Attempt to solve 3TQ M1

$x = 3, 6$ Both A1

[6]