STRAIGHT LINE COORDINATE GEOMETRY

4)

a) $A(-6, 4)$

$B(8, -3)$

L

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-4}{-3-4} = \frac{x--6}{8--6}$$

$$\frac{y-4}{-7} = \frac{x+6}{14}$$

$$14(y-4) = -7(x+6)$$

$$2(y-4) = -1(x+6)$$

$$2y - 8 = -x - 6$$

$$x + 2y - 2 = 0$$

---

b) $|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(8--6)^2 + (-3-4)^2}$

$= \sqrt{14^2 + 7^2}$

$= \sqrt{196 + 49}$

$= \sqrt{245}$

$= \sqrt{49 \times 5}$

$= 7\sqrt{5}$

STRAIGHT LINE COORDINATE GEOMETRY

10) Q(1, 3)

a) R(7, 0)

$$|QR| = \sqrt{(7-1)^2 + (0-3)^2}$$

$$= \sqrt{6^2 + 3^2} \quad = \sqrt{45} \quad = 3\sqrt{5}$$

---

b) Gradient $\ell_1 = \frac{3-0}{1-7} = \frac{3}{-6} = -\frac{1}{2}$

$\perp$ line ℓ_2 will have gradient $+2$

Passes through Q(1, 3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 2(x - 1)$$

$$y - 3 = 2x - 2$$

$$y = 2x + 1$$

---

c) At P, $x = 0$

$$y = 2(0) + 1 = 1$$

$\therefore$ P(0, 1)

---

d) $|QP| = \sqrt{(1-0)^2 + (3-1)^2} \quad = \sqrt{5}$

Area of $\triangle PQR = \frac{1}{2} \times |PQ| \times |QR|$ since $\angle PQR = 90^\circ$

$$= \frac{1}{2} \times \sqrt{5} \times 3\sqrt{5}$$

$$= \frac{15}{2} \text{ units}^2$$

STRAIGHT LINE COORDINATE GEOMETRY

10) A(2, 5) gradient $-\frac{1}{2}$ $\quad y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 2)$

$y - 5 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 6$

---

b) B(−2, 7) When $x = -2$, $\quad y = -\frac{1}{2}(-2) + 6$

$y = 1 + 6 = 7$

$\therefore (-2, 7)$ on line ℓ_1

---

c) $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - -2)^2 + (5 - 7)^2}$

$= \sqrt{4^2 + (-2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

---

d) If $x = p$, $y = -\frac{1}{2}p + 6$ $\quad C\left(p, -\frac{1}{2}p + 6\right)$

$|AC| = 5 = \sqrt{(p - 2)^2 + \left(-\frac{1}{2}p + 6 - 5\right)^2}$

squaring $\Rightarrow 25 = (p - 2)^2 + \left(-\frac{1}{2}p + 1\right)^2$

$25 = p^2 - 4p + 4 + \frac{1}{4}p^2 - p + 1$

$100 = 4p^2 - 16p + 16 + p^2 - 4p + 4$

$0 = 5p^2 - 20p - 80$

$0 = p^2 - 4p - 16$

STRAIGHT LINE COORDINATE GEOMETRY

8) a) A(6,7) B(8,2)

gradient AB $= \frac{7-2}{6-8} = -\frac{5}{2}$

$\perp$ line ℓ has gradient $+\frac{2}{5}$ and passes through A(6,7)

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{5}(x-6)$

$5y - 35 = 2(x-6)$

$5y - 35 = 2x - 12$

$2x - 5y + 23 = 0$

---

b) At C, $x = 0$

$2(0) - 5y + 23 = 0$

$23 = 5y$

$\frac{23}{5} = y$

$\therefore C\left(0, \frac{23}{5}\right)$

---

c) To find area of $\triangle OCB$, use OC as base and x coord of B as height

Area $= \frac{1}{2} \times \frac{23}{5} \times 8$

$= \frac{92}{5}$ units^2

STRAIGHT LINE COORDINATE GEOMETRY

3)

$\ell_1 \qquad 3x + 5y - 2 = 0$

a) $5y = -3x + 2$

$y = -\frac{3}{5}x + \frac{2}{5}$

gradient of $\ell_1 = -\frac{3}{5}$

b) $\perp$ line ℓ_2 has gradient $+\frac{5}{3}$ passes through $(3,1)$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{3}(x - 3)$

$y - 1 = \frac{5}{3}x - 5$

$y = \frac{5}{3}x - 4$

STRAIGHT LINE COORDINATE GEOMETRY

8) a) $A(7,4)$, $B(2,0)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \qquad \frac{y-4}{0-4} = \frac{x-7}{2-7}$$

$$\frac{y-4}{-4} = \frac{x-7}{-5}$$

$$-5(y-4) = -4(x-7)$$

$$-5y+20 = -4x+28$$

$$4x-5y-8=0$$

b) $|AB| = \sqrt{(7-2)^2+(4-0)^2} = \sqrt{5^2+4^2} = \sqrt{41}$

c) $C(2,t)$ $\quad |AC| = \sqrt{(7-2)^2+(4-t)^2} = \sqrt{5^2+(4-t)^2}$

If $|AC| = |AB|$ then $|AC|^2 = |AB|^2$

$$\Rightarrow 25+(4-t)^2 = 41$$

$$(4-t)^2 = 16$$

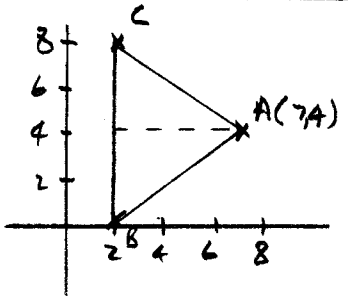
$$4-t = \pm 4$$

$$4 \pm 4 = t$$

$$t = 0 \text{ or } t = 8$$

$\therefore B(2,0)$ and $C(2,8)$

d)



Base $|BC| = 8$, height $= 7-2 = 5$

$$\text{Area} = \frac{1}{2} \times 8 \times 5$$

$$= 20 \text{ units}^2$$