

C2 EXPONENTIALS AND LOGARITHMS

1) a) $2 \log_3(x-5) - \log_3(2x-13) = 1$

$\log_3(x-5)^2 - \log_3(2x-13) = 1$

$\log_3\left(\frac{(x-5)^2}{(2x-13)}\right) = 1$

$\frac{(x-5)^2}{(2x-13)} = 3^1$

$x^2 - 10x + 25 = 3(2x-13)$

$x^2 - 10x + 35 = 6x - 39$

$x^2 - 10x - 6x + 35 + 39 = 0$

$x^2 - 16x + 64 = 0$

b) $\Rightarrow (x-8)^2 = 0$

$\Rightarrow x = 8$

2) a) $\log_x 64 = 2$

$\Rightarrow 64 = x^2$

$\Rightarrow x = 8$

b) $\log_2(11-6x) = 2 \log_2(x-1) + 3$

$\log_2(11-6x) = \log_2(x-1)^2 + 3$

$\log_2(11-6x) - \log_2(x-1)^2 = 3$

$\log_2\left(\frac{(11-6x)}{(x-1)^2}\right) = 3$

$\frac{11-6x}{(x-1)^2} = 2^3$

$11-6x = 8(x-1)^2$

$11-6x = 8(x^2 - 2x + 1)$

$11-6x = 8x^2 - 16x + 8$

$0 = 8x^2 - 10x - 3$

$0 = (4x+1)(2x-3)$

$\Rightarrow 4x+1=0$ or $2x-3=0$

$4x = -1$

$2x = 3$

~~$x = -\frac{1}{4}$~~

$x = \frac{3}{2}$

ignore since $\ln(x-1)$ not defined for $x = -\frac{1}{4}$

$x = \frac{3}{2}$

3) $\log_5(4-x) - 2 \log_5 x = 1$

$\log_5(4-x) - \log x^2 = 1$

$\log_5\left(\frac{4-x}{x^2}\right) = 1$

$\frac{4-x}{x^2} = 5^1$

$4-x = 5x^2$

$0 = 5x^2 + x - 4$

$0 = (5x-4)(x+1)$

$\Rightarrow 5x-4=0$ or $x+1=0$

$5x = 4$

~~$x = -1$~~

$x = \frac{4}{5}$

ignore since $0 < x < 4$

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4) $a = 3b$ ①

$$\log_3 a + \log_3 b = 2$$

$$\log_3(ab) = 2$$

$$ab = 3^2$$

$$ab = 9$$
 ②

Sub for a in ②

$$(3b)b = 9$$

$$b^2 = 3$$

$$\underline{b = \sqrt{3}} \quad (\text{cannot have } \ln(-\sqrt{3}))$$

$$\underline{a = 3\sqrt{3}}$$

5) i) $\log_6 36$

$$36 = 6^2 \Rightarrow \log_6 36 = 2$$

ii) $2\log_a 3 + \log_a 11$

$$= \log_a 3^2 + \log_a 11$$

$$= \log_a 9 + \log_a 11$$

$$= \log_a(9 \times 11)$$

$$= \log_a(99)$$

6) a) $5^x = 8$

$$\ln 5^x = \ln 8$$

$$x \ln 5 = \ln 8$$

$$x = \frac{\ln 8}{\ln 5}$$

$$\underline{x = 1.29 \text{ to 3 s.f.}}$$

b) $\log_2(x+1) - \log_2 x = \log_2 7$

$$\log_2\left(\frac{x+1}{x}\right) = \log_2 7$$

$$\Rightarrow \left(\frac{x+1}{x}\right) = 7$$

$$x+1 = 7x$$

$$1 = 7x - x$$

$$1 = 6x$$

$$x = \frac{1}{6}$$

7) $3^x = 5$

a) $\ln 3^x = \ln 5$

$$x \ln 3 = \ln 5$$

$$x = \frac{\ln 5}{\ln 3}$$

$$\underline{x = 1.46 \text{ to 3 s.f.}}$$

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7) b) $\log_2(2x+1) - \log_2 x = 2$

$\log_2\left(\frac{2x+1}{x}\right) = 2$

$\frac{2x+1}{x} = 2^2$

$2x+1 = 4x$

$1 = 4x - 2x$

$1 = 2x$

$x = \frac{1}{2}$

$\Rightarrow 2a - b = 1$ (1)

$\Rightarrow 2 + a + \frac{1}{2}b = 1$ (2)

d) From (1) $2a = 1 + b$

$2a - 1 = b$

Sub for b in (2)

$2 + a + \frac{1}{2}(2a - 1) = 1$

$2 + a + a - \frac{1}{2} = 1$

$2a + \frac{3}{2} = 1$

$2a = 1 - \frac{3}{2}$

$2a = -\frac{1}{2}$

$a = -\frac{1}{4}$

From (1) $b = 2a - 1$

$b = 2(-\frac{1}{4}) - 1$

$b = -\frac{1}{2} - 1$

$b = -\frac{3}{2}$

8) a) $\log_5 x = a, \log_5 y = b$

$\log_5\left(\frac{x^2}{y}\right) = \log_5 x^2 - \log_5 y$

$= 2\log_5 x - \log_5 y$

$= 2a - b$

b) $\log_5(25x\sqrt{y})$

$= \log_5 25 + \log_5 x + \log_5 y^{\frac{1}{2}}$

$= 2 + a + \frac{1}{2}\log_5 y$

$= 2 + a + \frac{1}{2}b$

e) $x = 5^{-\frac{1}{4}} = 0.669$

$y = 5^{-\frac{3}{2}} = 0.089$

to 3 d.p.

c) $\log_5\left(\frac{x^2}{y}\right) = 1$

$\log_5(25x\sqrt{y}) = 1$

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$$9) \log_2 x = a$$

$$\begin{aligned} a) \log_2(16x) &= \log_2 16 + \log_2 x \\ &= 4 + a \end{aligned}$$

$$\begin{aligned} b) \log_2\left(\frac{x^4}{2}\right) \\ &= \log_2 x^4 - \log_2 2 \\ &= 4 \log_2 x - 1 \\ &= 4a - 1 \end{aligned}$$

$$c) \log_2(16x) - \log_2\left(\frac{x^4}{2}\right) = \frac{1}{2}$$

$$4 + a - (4a - 1) = \frac{1}{2}$$

$$4 + a - 4a + 1 = \frac{1}{2}$$

$$-3a + 5 = \frac{1}{2}$$

$$5 - \frac{1}{2} = 3a$$

$$\frac{9}{2} = 3a$$

$$\frac{3}{2} = a$$

$$\Rightarrow \log_2 x = \frac{3}{2}$$

$$\begin{aligned} \Rightarrow x &= 2^{3/2} \\ &= 2^1 \times 2^{1/2} \\ &= 2\sqrt{2} \end{aligned}$$

$$10) a) \frac{x^2 + 4x + 3}{x^2 + x}$$

$$= \frac{(x+1)(x+3)}{x(x+1)}$$

$$= \frac{x+3}{x}$$

$$b) \log_2(x^2 + 4x + 3) - \log_2(x^2 + x) = 4$$

$$\log_2\left(\frac{x^2 + 4x + 3}{x^2 + x}\right) = 4$$

$$\frac{x+3}{x} = 2^4$$

$$x+3 = 16x$$

$$3 = 15x$$

$$x = \frac{1}{5}$$

$$11) 2\log_3 x - \log_3(x-2) = 2$$

$$\log_3 x^2 - \log_3(x-2) = 2$$

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

$$\frac{x^2}{x-2} = 3^2$$

$$x^2 = 9(x-2)$$

$$x^2 = 9x - 18$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 3$$