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2) $\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx$

$= \frac{8x^4}{4} + \frac{6x^{3/2}}{3/2} - 5x + c$

$= 2x^4 + 4x^{3/2} - 5x + c$

---

7) $y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x}$ $\quad x > 0$

$y = 8x^3 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$

$\frac{dy}{dx} = 24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$

---

11) a) $\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$

$\frac{dy}{dx} = 3x - 5x^{-\frac{1}{2}} - 2$

$\Rightarrow \quad y = \frac{3x^2}{2} - \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + c$

$y = \frac{3x^2}{2} - 10x^{\frac{1}{2}} - 2x + c$

$P(4,5)$ on curve $\qquad 5 = \frac{3(4)^2}{2} - 10(4)^{\frac{1}{2}} - 2(4) + c$

$5 = 24 - 20 - 8 + c$

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11a cont)

$$9 = c$$

$$f(x) = y = \frac{3x^2}{2} - 10x^{\frac{1}{2}} - 2x + 9$$

---

b) When $x = 4$,

$$\frac{dy}{dx} = 3(4) - \frac{5}{\sqrt{4}} - 2$$

$$\frac{dy}{dx} = 12 - \frac{5}{2} - 2 = \frac{15}{2}$$

Tgt through $(4, 5)$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{15}{2}(x - 4)$$

$$2y - 10 = 15(x - 4)$$

$$2y - 10 = 15x - 60$$

$$0 = 15x - 2y - 50$$

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3)

a) $y = x^2 - k\sqrt{x}$

$y = x^2 - kx^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x - \frac{k}{2}x^{-\frac{1}{2}}$

---

b) when $x = 4$, $\frac{dy}{dx} < 0$

$\Rightarrow \quad 2x - \frac{k}{2\sqrt{x}} < 0$

$\Rightarrow \quad 2(4) - \frac{k}{2\sqrt{4}} < 0$

$\Rightarrow \quad 8 - \frac{k}{4} < 0$

$\Rightarrow \quad 32 - k < 0$

$\Rightarrow \quad 32 < k$

$k > 32$

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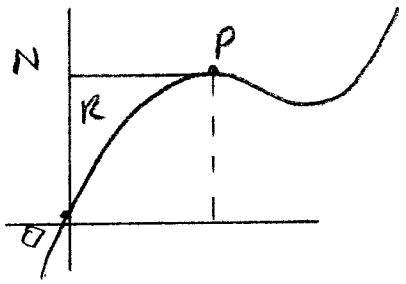
8) a)



$$y = x^3 - 10x^2 + kx$$

$$\frac{dy}{dx} = 3x^2 - 20x + k$$

when $x = 2$, $\frac{dy}{dx} = 0$ $\Rightarrow$ $3(2)^2 - 20(2) + k = 0$

$\Rightarrow$ $12 - 40 + k = 0$

$\Rightarrow$ $-28 + k = 0$

$\Rightarrow$ $k = 28$

---

b) Region R Area = Area of rectangle − Area under curve

when $x = 2$, $y = 2^3 - 10(2)^2 + 28(2) = 24$ so $P(2, 24)$

$$\text{Area of } R = 24 \times 2 - \int_0^2 (x^3 - 10x^2 + 28x)\, dx$$

$$= 48 - \left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2 \right]_0^2$$

$$= 48 - \left[\left(\frac{2^4}{4} - \frac{10(2)^3}{3} + 14(2)^2 \right) - (0) \right]$$

$$= 48 - \frac{100}{3}$$

$$= \frac{44}{3} \text{ units}^2$$
