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3) a) $y = 2x^3 + \frac{3}{x^2}$ $\quad x \neq 0$

$y = 2x^3 + 3x^{-2}$

$\frac{dy}{dx} = 6x^2 - 6x^{-3}$

---

b) $\int (2x^3 + 3x^{-2})\,dx$

$= \frac{2x^4}{4} + \frac{3x^{-1}}{-1} + c$

$= \frac{1}{2}x^4 - \frac{3}{x} + c$

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9)

a) $f(x) = \dfrac{(3-4\sqrt{x})^2}{\sqrt{x}} \qquad x > 0$

$$= \frac{9 - 24\sqrt{x} + 16x}{\sqrt{x}}$$

$$f(x) = 9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$$

---

b) $$f'(x) = -\tfrac{1}{2} \times 9x^{-3/2} + \tfrac{1}{2} \times 16x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{9}{2}x^{-3/2} + 8x^{-\frac{1}{2}}$$

---

c) $$f'(x) = \frac{-9}{2x^{3/2}} + \frac{8}{\sqrt{x}}$$

$$f'(9) = \frac{-9}{2(9)^{3/2}} + \frac{8}{\sqrt{9}}$$

$$= \frac{-9}{54} + \frac{8}{3}$$

$$= -\frac{1}{6} + \frac{8}{3}$$

$$= \frac{15}{6}$$

$$= \frac{5}{2}$$

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11) a) $y = x^3 - 2x^2 - x + 9$ $\quad x > 0$

$P(2,7)$

when $x = 2$, $y = 2^3 - 2(2)^2 - 2 + 9$

$= 8 - 8 - 2 + 9$

$= 7$ $\quad \therefore P(2,7)$ on C

---

b) $\frac{dy}{dx} = 3x^2 - 4x - 1$

when $x = 2$, $\frac{dy}{dx} = 3(2)^2 - 4(2) - 1 = 3$

Tgt gradient $= 3$, through $(2,7)$

$y - y_1 = m(x - x_1)$

$y - 7 = 3(x - 2)$

$y - 7 = 3x - 6$

$y = 3x + 1$

---

c) Gradient at Q $= -\frac{1}{3}$

$3x^2 - 4x - 1 = -\frac{1}{3}$

$9x^2 - 12x - 3 = -1$

$9x^2 - 12x - 2 = 0$

$x = \frac{12 \pm \sqrt{144 + 72}}{18} = \frac{12 \pm \sqrt{216}}{18} = \frac{12 \pm 6\sqrt{6}}{18}$

$x = \frac{2 \pm \sqrt{6}}{6}$ but $x > 0$ $\quad \therefore x = \frac{2 + \sqrt{6}}{3}$

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1) $\int_1^4 (2x + 3\sqrt{x})\,dx$

$= \int_1^4 (2x + 3x^{\frac{1}{2}})\,dx$

$= \left[\frac{2x^2}{2} + \frac{3x^{3/2}}{3/2} \right]_1^4$

$= \left[x^2 + 2x^{3/2} \right]_1^4$

$= \left(4^2 + 2(4)^{3/2} \right) - \left(1^2 + 2(1)^{3/2} \right)$

$= (16 + 16) - (1 + 2)$

$= 29$

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9)

a) $\text{Volume} = \frac{1}{2}r^2\theta h \Rightarrow \frac{1}{2}r^2 \times 1 \times h = 300$

$r^2h = 600$

$h = \frac{600}{r^2}$

Surface area =

$rh + rh + r\theta h + \frac{1}{2}r^2\theta + \frac{1}{2}r^2\theta$

$= \frac{600}{r} + \frac{600}{r} + \frac{600 \times 1}{r} + \frac{1}{2}r^2 \times 1 + \frac{1}{2}r^2 1$

$= \frac{1800}{r} + r^2$

---

b) $S = 1800r^{-1} + r^2$

$\frac{dS}{dr} = -1800r^{-2} + 2r = 2r - \frac{1800}{r^2}$

S stationary when $\frac{dS}{dr} = 0 \Rightarrow 2r - \frac{1800}{r^2} = 0$

$\Rightarrow 2r^3 - 1800 = 0$

$\Rightarrow r^3 = 900$

$\Rightarrow r = \sqrt[3]{900} = 9.65\text{ cm}$

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c) $\frac{d^2S}{dr^2} = 2 + 3600r^{-3} = 2 + \frac{3600}{r^3} > 0$ when $r = 9.65$

$\therefore$ a minimum

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d) $\text{Min } S = \frac{1800}{9.65} + 9.65^2 = 280\text{ cm}^2$ to 3 s.f.
