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1) $y = x^4 + x^{\frac{1}{3}} + 3 \qquad \frac{dy}{dx} = 4x^3 + \frac{1}{3}x^{-2/3}$

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4) $\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x} \qquad x > 0$

$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x^{3/2}$$

$$\Rightarrow y = \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{5/2}}{\frac{5}{2}} + c$$

$$y = 10x^{\frac{1}{2}} + \frac{2x^{5/2}}{5} + c$$

Given $y = 35, x = 4$

$$35 = 10(4)^{\frac{1}{2}} + \frac{2(4)^{\frac{5}{2}}}{5} + c$$

$$35 = 20 + \frac{64}{5} + c$$

$$\frac{11}{5} = c$$

$$y = 10x^{\frac{1}{2}} + \frac{2x^{5/2}}{5} + \frac{11}{5}$$

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6) a) $y = \frac{(x+3)(x-8)}{x} = \frac{x^2+3x-8x-24}{x}$

$y = x - 11 - 24x^{-1}$

$\frac{dy}{dx} = 1 + 24x^{-2} = 1 + \frac{24}{x^2}$

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b) when $x = 2$, $y = \frac{(2+3)(2-8)}{2} = \frac{5(-6)}{2} = -15$

when $x = 2$, $\frac{dy}{dx} = 1 + \frac{24}{2^2} = 1 + 6 = 7$

Tgt $y - y_1 = m(x - x_1)$

$y - -15 = 7(x-2)$

$y + 15 = 7x - 14$

$y = 7x - 29$

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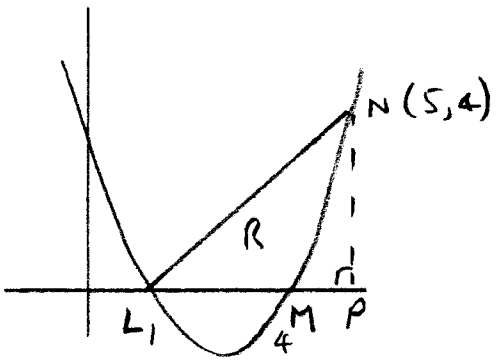
7) a)



$y = x^2 - 5x + 4$

At L, M $y = 0$

$\Rightarrow x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1$ or $x = 4$

L(1,0) M(4,0)

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7) b) When $x = 5$, $y = 5^2 - 5(5) + 4$

$= 25 - 25 + 4$

$= 4$

so (5,4) on curve

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c) $\int (x^2 - 5x + 4)\,dx = \frac{x^3}{3} - \frac{5x^2}{2} + 4x + C$

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d) Area of R = Area of $\triangle LMP$ − Area under curve from 4 to 5

(See previous diagram for P)

Area of R $= \frac{1}{2} \times 4 \times 4 - \int_4^5 (x^2 - 5x + 4)\,dx$

$= 8 - \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x \right]_4^5$

$= 8 - \left[\left(\frac{5^3}{3} - \frac{5(5)^2}{2} + 4(5) \right) - \left(\frac{4^3}{3} - \frac{5(4)^2}{2} + 4(4) \right) \right]$

$= 8 - \left[\frac{11}{6} \right]$

$= \frac{37}{6}$

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9) $y = 12\sqrt{x} - x^{3/2} - 10 \qquad x > 0$

a) $y = 12x^{\frac{1}{2}} - x^{3/2} - 10$

$\frac{dy}{dx} = \frac{1}{2} \times 12x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

At t.p. $\frac{dy}{dx} = 0 \Rightarrow 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

Multiply by $x^{\frac{1}{2}}$ $\qquad 6 - \frac{3}{2}x = 0$

$6 = \frac{3}{2}x$

$12 = 3x$

$x = 4$

$y = 12\sqrt{4} - 4^{3/2} - 10$

$y = 24 - 8 - 10 = 6$

Turning point at $(4, 6)$

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b) $\frac{d^2y}{dx^2} = -\frac{1}{2} \times 6x^{-3/2} - \frac{1}{2} \times \frac{3}{2}x^{-\frac{1}{2}}$

$= -3x^{-3/2} - \frac{3}{4}x^{-\frac{1}{2}}$

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c) $\frac{d^2y}{dx^2} < 0$ for $x > 0$ $\therefore$ a maximum

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