Leave blank

8. The weight, X grams, of soup put in a tin by machine A is normally distributed with a mean of 160 g and a standard deviation of 5 g.
A tin is selected at random.

$X \sim N(\overset{\mu}{160}, \overset{\sigma^2}{5^2})$

(a) Find the probability that this tin contains more than 168 g.
(3)

The weight stated on the tin is w grams.

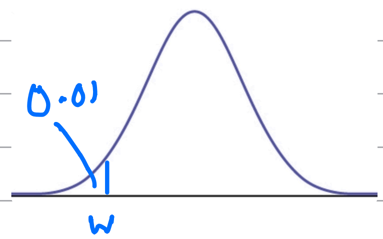
(b) Find w such that $P(X < w) = 0.01$
(3)

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams.

(c) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$ find the value of μ and the value of σ.
(6)

a) $P(X > 168) = 0.0548$

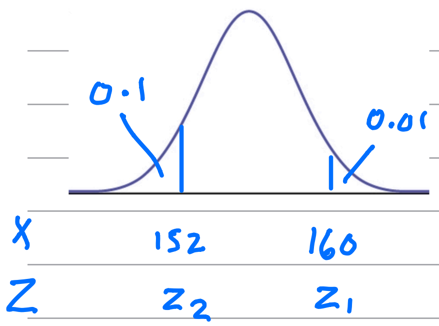
b)



$W = 148.4$

INVERSE NORMAL
Area = 0.01
$\sigma = 5$
$\mu = 160$

c) $z = \dfrac{x - \mu}{\sigma}$ $\quad \sigma z = x - \mu \quad \sigma z + \mu = x$



$z_1 = \Phi^{-1}(0.99) = 2.326$

$z_2 = \Phi^{-1}(0.1) = -1.282$

$2.326\sigma + \mu = 160$

$-1.282\sigma + \mu = 152$

By calculator $\quad \sigma = 2.217, \quad \mu = 154.8$

Leave blank

2. The random variable $X \sim \mathrm{N}(\mu, 5^2)$ and $\mathrm{P}(X < 23) = 0.9192$

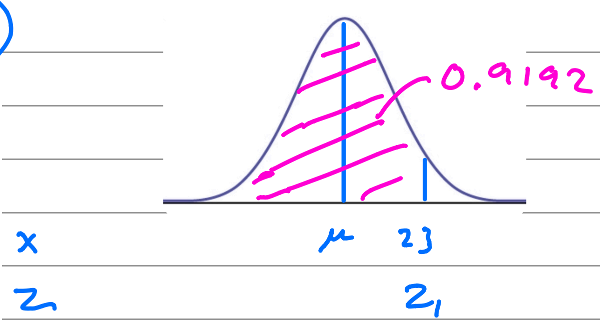
(a) Find the value of μ.

(4)

(b) Write down the value of $\mathrm{P}(\mu < X < 23)$.

(1)

a)



$z_1 = \Phi^{-1}(0.9192)$

$z_1 = 1.3997$

$Z = \dfrac{X - \mu}{\sigma} \Rightarrow \sigma Z = X - \mu$

$\Rightarrow \mu = X - \sigma Z$

$\mu = 23 - 5 \times 1.3997$

$\mu = 16.00$

b) $P(\mu < X < 23) = P(X < 23) - P(X < \mu)$

$= 0.9192 - 0.5$

$= 0.4192$

Leave blank

4. Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

$X \sim N(\overset{\mu}{16.12}, \overset{\sigma^2}{1.60^2})$

A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s.

(3)

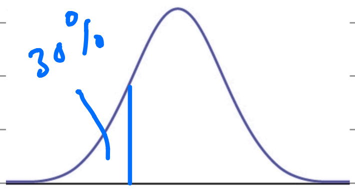
On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

(b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.

(4)

a) $P(X < 15) = 0.2420$

b)



z $\quad z_1$

x $\quad 15.28$

Fastest 30% have smallest times

$z_1 = \Phi^{-1}(0.3) = -0.5244$

$Z = \dfrac{X - \mu}{\sigma} \Rightarrow X = \sigma Z + \mu$

$X = -0.5244 \times 1.60 + 16.12$

$X = 15.28\,s$ to 2 d.p.

P 3 8 1 6 4 A 0 8 2 4

Leave blank

7. A manufacturer fills jars with coffee. The weight of coffee, W grams, in a jar can be modelled by a normal distribution with mean 232 grams and standard deviation 5 grams.

$$X \sim N(\overset{\mu}{232}, \overset{\sigma^2}{5^2})$$

(a) Find $P(W < 224)$. **(3)**

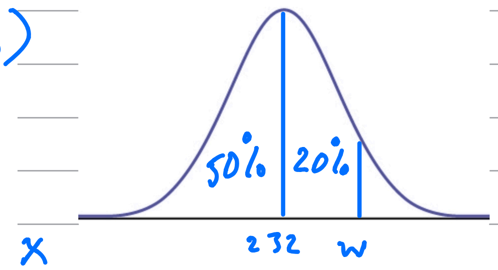
(b) Find the value of w such that $P(232 < W < w) = 0.20$ **(4)**

Two jars of coffee are selected at random.

(c) Find the probability that only one of the jars contains between 232 grams and w grams of coffee. **(3)**

a) $P(W < 224) = 0.0548$

b)



Area $= 0.7$

$\underline{w = 234.6}$

c) P(one in between 232 and w and one not)

$= 2 \times 0.2 \times 0.8$

$= 0.32$

Multiply by 2 since could occur in either order.

P 4 0 6 9 9 A 0 2 2 2 4

Leave blank

6. The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm.

$X \sim N(\overset{\mu}{162}, \overset{\sigma^2}{7.5^2})$

(a) Find the probability that a randomly chosen adult female is taller than 150 cm.
(3)

Sarah is a young girl. She visits her doctor and is told that she is at the 60th percentile for height.

(b) Assuming that Sarah remains at the 60th percentile, estimate her height as an adult.
(3)

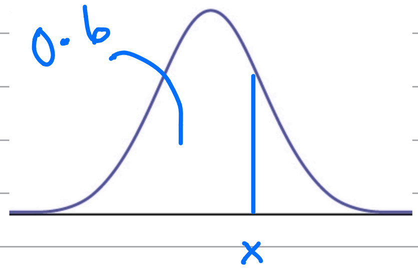
The heights of an adult male population are normally distributed with standard deviation 9.0 cm.

Given that 90% of adult males are taller than the mean height of adult females,

(c) find the mean height of an adult male.
(4)

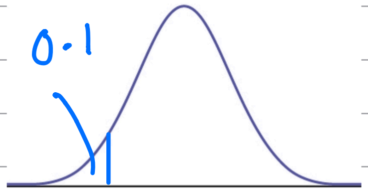
a) $P(X > 150) = 0.9452$

b)



Estimate $= 163.9$ cm

c) $Y \sim N(\mu, 9.0^2)$ 90% taller than 162 cm



X 162

Z z_1

$z_1 = \Phi^{-1}(0.1) = -1.2816$

$z = \dfrac{X - \mu}{\sigma}$ $\sigma z = X - \mu$

$\mu = X - \sigma z$

$\mu = 162 + 9 \times 1.2816$

$\mu = 173.5$ cm

P 4 0 1 0 5 X A 0 1 8 2 4

4. The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

$L \sim N(\overset{\mu}{100}, \overset{\sigma^2}{15^2})$

(a) Find $P(L > 127)$.
(3)

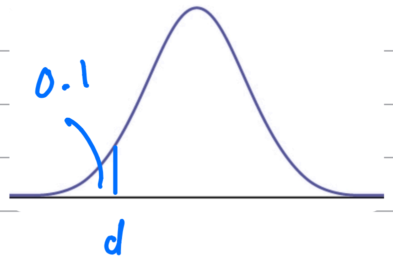
(b) Find the value of d such that $P(L < d) = 0.10$
(3)

Alice is about to go on a 6 hour journey.
Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.
(4)

a) $P(L > 127) = \underline{0.0354}$

b)



$\underline{d = 80.78}$

c) Assuming phone working after 127 hours

$P(L > 133 \mid L > 127)$

$$= \frac{P(L > 133 \cap L > 127)}{P(L > 127)}$$

$$= \frac{P(L > 133)}{P(L > 127)}$$

$$= \frac{0.0139}{0.0359}$$

$$= \underline{0.3872}$$

Leave blank

6. The weight, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8

$X \sim N(\mu, 7.8^2)$

Given that 10% of tins contain less than 200 g, find

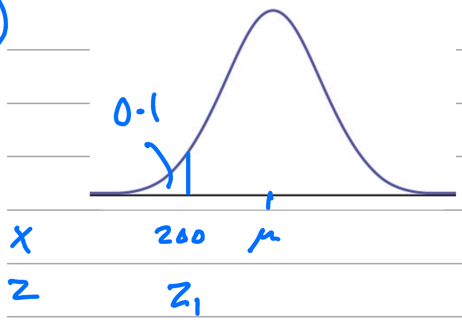
(a) the value of μ **(3)**

(b) the percentage of tins that contain more than 225 g of beans. **(3)**

The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation σ.

(c) Given that 98% of tins contain between 200 g and 210 g find the value of σ. **(4)**

a)



$z_1 = \Phi^{-1}(0.1) = -1.2816$

$z = \dfrac{x-\mu}{\sigma} \Rightarrow \sigma z = x - \mu$

$\Rightarrow \mu = x - \sigma z$

$\mu = 200 + 7.8 \times 1.2816$

$\underline{\mu = 210.0 \text{ g}}$

b) $X \sim N(210, 7.8^2)$ $\quad P(X > 225) = 0.0272$

so $\underline{2.72\% \text{ contain more than } 225\text{g}}$

c) $Y \sim N(205, \sigma^2)$

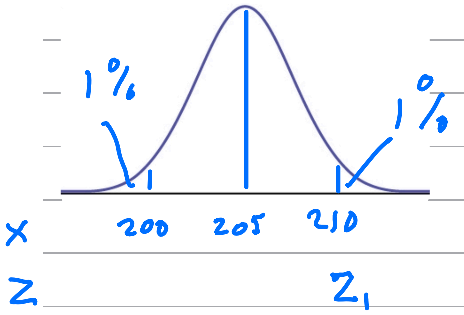
200 and 210 symmetrical about $\mu = 205$

so 1% at each end

P 4 2 8 3 1 A 0 2 2 2 4

Leave blank

Question 6 continued



$$z_1 = \Phi^{-1}(0.99) = 2.3263$$

$$z = \frac{x-\mu}{\sigma}$$

$$\sigma z = x - \mu$$

$$\sigma = \frac{x-\mu}{z}$$

$$\sigma = \frac{210-205}{2.3263}$$

$$\sigma = 2.149$$

Q6

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END