

## Integration By Parts

### Exercise 11F

1c)  $\int x \sec^2 x \, dx$

Let  $u = x$    Let  $\frac{du}{dx} = \sec^2 x$   
 $\Rightarrow \frac{du}{dx} = 1$     $\Rightarrow v = \tan x$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx$$

$$= x \tan x - \int \frac{\sin x}{\cos x} \, dx$$

$$= x \tan x + \ln|\cos x| + C$$


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2 a)  $\int \ln x \, dx = \int 1 \ln x \, dx$

Let  $u = \ln x$    Let  $\frac{du}{dx} = 1$   
 $\Rightarrow \frac{du}{dx} = \frac{1}{x}$     $\Rightarrow v = x$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\begin{aligned} \int \ln x \, dx &= x \ln x - \int x \cdot \frac{1}{x} \, dx \\ &= x \ln x - \int 1 \, dx \\ &= x \ln x - x + C \end{aligned}$$


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$$2a) \int (\ln x)^2 dx = \int \ln x \times \ln x dx$$

$$\begin{aligned} \text{Let } u &= \ln x & \text{Let } \frac{du}{dx} &= \ln x \\ \Rightarrow \frac{du}{dx} &= \frac{1}{x} & \Rightarrow v &= x \ln x - x \end{aligned}$$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\begin{aligned} \int (\ln x)^2 dx &= (x \ln x - x) \ln x - \int (x \ln x - x) \frac{1}{x} dx \\ &= (x \ln x - x) \ln x - \int (\ln x - 1) dx \\ &= (x \ln x - x) \ln x - x \ln x + x + x + C \\ &= (x \ln x - x) \ln x - x \ln x + 2x + C \\ &= x (\ln x)^2 - 2x \ln x + 2x + C \end{aligned}$$

$$4a) \int_0^{\ln 2} x e^{2x} dx$$

$$\begin{aligned} \text{Let } u &= x & \text{Let } \frac{du}{dx} &= e^{2x} \\ \Rightarrow \frac{du}{dx} &= 1 & \Rightarrow v &= \frac{1}{2} e^{2x} \end{aligned}$$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int_0^{\ln 2} x e^{2x} dx = \left[ \frac{1}{2} x e^{2x} \right]_0^{\ln 2} - \int_0^{\ln 2} \frac{1}{2} e^{2x} dx$$

$$= \left[ \frac{1}{2} x e^{2x} \right]_0^{\ln 2} - \left[ \frac{1}{4} e^{2x} \right]_0^{\ln 2}$$

$$= \left[ \frac{1}{2} \ln 2 e^{2\ln 2} - 0 \right] - \left[ \frac{1}{4} e^{2\ln 2} - \frac{1}{4} e^0 \right]$$

$$= \frac{1}{2} \ln 2 e^{\ln 4} - \frac{1}{4} e^{\ln 4} + \frac{1}{4}$$

$$= 2 \ln 2 - 1 + \frac{1}{4}$$

$$= 2 \ln 2 - \frac{3}{4}$$


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3d)  $\int 2x^2 \sin 2x \, dx$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int 2x^2 \sin 2x \, dx = -x^2 \cos 2x + \int 2x \cos 2x \, dx$$

Let  $u = 2x^2$     Let  $\frac{du}{dx} = \sin 2x$   
 $\Rightarrow \frac{du}{dx} = 4x$      $\Rightarrow v = -\frac{1}{2} \cos 2x$

Aside  $\int 2x \cos 2x \, dx$     Let  $u = 2x$     Let  $\frac{du}{dx} = \cos 2x$   
 $\Rightarrow \frac{du}{dx} = 2$      $\Rightarrow v = \frac{1}{2} \sin 2x$

$$= x \sin 2x - \int \sin 2x \, dx$$

$$= x \sin 2x + \frac{1}{2} \cos 2x + C$$

$$\int 2x^2 \sin 2x \, dx = -x^2 \cos 2x + x \sin 2x + \frac{1}{2} \cos 2x + C$$

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