

Converting Parametrics to Cartesian Form

(P) 5 Find the Cartesian equation of the curves given by the following parametric equations.

a) $x = \sin t, \quad y = \sin\left(t + \frac{\pi}{4}\right), \quad \frac{\pi}{4} < t < \pi$

b) $x = 3 \cos t, \quad y = 2 \cos\left(t + \frac{\pi}{6}\right), \quad 0 < t < \frac{\pi}{3}$

c) $x = \sin t, \quad y = 3 \sin(t + \pi), \quad 0 < t < 2\pi$

Hint

Use the addition formulae
and exact values.

a) $x = \sin t \quad y = \sin t \cos \frac{\pi}{4} + \cos t \sin \frac{\pi}{4}$

$$y = \frac{1}{\sqrt{2}} \sin t + \frac{1}{\sqrt{2}} \cos t$$

$$y = \frac{1}{\sqrt{2}} (\sin t + \sqrt{1 - \sin^2 t})$$

$$y = \frac{1}{\sqrt{2}} (x + \sqrt{1 - x^2})$$

b) $x = 3 \cos t \quad y = 2 \left(\cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6} \right)$

$$\frac{x}{3} = \cos t \quad y = 2 \left(\frac{\sqrt{3}}{2} \cos t - \frac{1}{2} \sin t \right)$$

$$y = \sqrt{3} \cos t - \sin t$$

$$y = \sqrt{3} \cos t - (\sqrt{1 - \cos^2 t})$$

$$y = \sqrt{3} \frac{x}{3} - \sqrt{1 - \frac{x^2}{9}}$$

c) $x = \sin t \quad y = 3 \sin(t + \pi)$

$$y = 3(\sin t \cos \pi + \cos t \sin \pi)$$

$$y = -3 \sin t + 0$$

$$y = -3x$$

1  $(\sin(t), 3 \sin(t + \pi))$ X

0 $\leq t \leq$ 2π

