

3. Using the substitution  $u^2 = 2x - 1$ , or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}} dx. \quad (8)$$

$$\begin{aligned}
 &= \int_1^3 \frac{3\left(\frac{u^2+1}{2}\right)u du}{u} \\
 &= \frac{3}{2} \int_1^3 (u^2 + 1) du \\
 &= \frac{3}{2} \left[ \frac{u^3}{3} + u \right]_1^3 \\
 &= \frac{3}{2} \left[ \left( \frac{3^3}{3} + 3 \right) - \left( \frac{1^3}{3} + 1 \right) \right] \\
 &= \frac{3}{2} \left[ 12 - \frac{4}{3} \right] \\
 &= 16
 \end{aligned}$$

Let  $u^2 = 2x - 1$   
 $2u \frac{du}{dx} = 2$   
 $u du = dx$   
 Also  $\frac{u^2 + 1}{2} = x$   
 limits  $x = 5 \quad u = 3$   
 $x = 1 \quad u = 1$

4. (i) Find  $\int \ln\left(\frac{x}{2}\right) dx.$  (4)

(ii) Find the exact value of  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x dx$  (5)

ii)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 - \cos 2x) \, dx$$

$$= \frac{1}{2} \left[ x - \frac{1}{2} \sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left[ \left( \frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left( \frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} \right) \right]$$

$$= \frac{1}{2} \left[ \frac{\pi}{2} - 0 - \left( \frac{\pi}{4} - \frac{1}{2} \right) \right]$$

$$= \frac{1}{2} \left[ \frac{\pi}{4} + \frac{1}{2} \right]$$

.....0003443

$$= \frac{\pi}{8} + \frac{1}{4} \quad \text{or} \quad \frac{\pi + 2}{8}$$


---

i)  $\int \ln \left( \frac{x}{2} \right) \, dx$  needs integration by parts

4. Use the substitution  $x = \sin \theta$  to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx. \quad (7)$$

$$\begin{aligned}
 &= \int_0^{\frac{\pi}{6}} \frac{1}{(1-\sin^2 \theta)^{\frac{3}{2}}} \cos \theta d\theta \\
 &= \int_0^{\frac{\pi}{6}} \frac{1}{\cos^3 \theta} \cos \theta d\theta \\
 &= \int_0^{\frac{\pi}{6}} \frac{1}{\cos^2 \theta} d\theta \\
 &= \int_0^{\frac{\pi}{6}} \sec^2 \theta d\theta \\
 &= \left[ \tan \theta \right]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} - 0 \\
 &= \frac{1}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}}{3}
 \end{aligned}$$

Let  $x = \sin \theta$   
 $\frac{dx}{d\theta} = \cos \theta$   
 $dx = \cos \theta d\theta$   
 Limits  
 $x = \frac{1}{2} \quad \theta = \frac{\pi}{6}$   
 $x = 0 \quad \theta = 0$

2. Use the substitution  $u = 2^x$  to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2} dx. \quad (6)$$

$$\begin{aligned}
 &= \int_1^2 \frac{u}{(u+1)^2} \frac{du}{u \ln 2} \\
 &= \frac{1}{\ln 2} \int_1^2 \frac{1}{(u+1)^2} du \\
 &= \frac{1}{\ln 2} \int_1^2 (u+1)^{-2} du \\
 &= \left[ -\frac{1}{\ln 2} (u+1)^{-1} \right]_1^2 \\
 &= -\frac{1}{\ln 2} \left[ \frac{1}{2+1} - \frac{1}{1+1} \right] \\
 &= -\frac{1}{6 \ln 2}
 \end{aligned}$$

$u = 2^x$   
 $\ln u = \ln 2^x$   
 $\ln u = x \ln 2$   
 $\frac{1}{u} \frac{du}{dx} = \ln 2$   
 $\frac{du}{dx} = u \ln 2$   
 $\frac{du}{u \ln 2} = dx$

limits  $x=1 \quad u=2$   
 $x=0 \quad u=1$

Note  $\frac{d}{dx} a^x = a^x \times \ln a$

Proof

$$\begin{aligned}
 y &= a^x \\
 \ln y &= \ln a^x \\
 \ln y &= x \ln a
 \end{aligned}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln a$$

$$\frac{dy}{dx} = y \ln a$$

$$\frac{dy}{dx} = a^x \times \ln a$$

---

Hwk for Tuesday 10<sup>th</sup> Dec

Watch Integration by Parts Video  
Print off Lesson Notes

Do and self mark associated exercise

---