

Differentiation

Rates of Change

Differentiation - Rates of Change

- 4 Water flows into a bowl at a constant rate of $10\text{ cm}^3\text{ s}^{-1}$ (see Fig. 4).

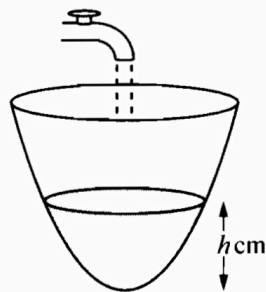


Fig. 4

When the depth of water in the bowl is $h\text{ cm}$, the volume of water is $V\text{ cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm . [5]

V volume h height t time

$$V = \pi h^2 \quad \frac{dV}{dh} = 2\pi h$$

Find $\frac{dh}{dt}$ when $h = 5\text{cm}$

$$\begin{aligned} \frac{dh}{dt} &= \frac{dh}{dV} \times \frac{dV}{dt} \\ &= \frac{1}{\frac{dV}{dh}} \times \frac{dV}{dt} \end{aligned}$$

$$\frac{dh}{dt} = \frac{1}{2\pi h} \times 10$$

$$\begin{aligned} \text{When } h = 5\text{cm} \quad \frac{dh}{dt} &= \frac{1}{2\pi \times 5} \times 10 \text{ cm s}^{-1} \\ &= \frac{1}{\pi} \text{ cm s}^{-1} \\ &= 0.318 \text{ cm s}^{-1} \end{aligned}$$
