

- 2 Given that  $y = 6x^{\frac{3}{2}}$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .

Show, without using a calculator, that when  $x = 36$  the value of  $\frac{d^2y}{dx^2}$  is  $\frac{3}{4}$ . [5]

$$y = 6x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2} \times 6x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2} \times 9x^{-\frac{1}{2}} = \frac{9}{2}x^{-\frac{1}{2}} = \frac{9}{2\sqrt{x}}$$

when  $x = 36$

$$\frac{d^2y}{dx^2} = \frac{9}{2\sqrt{36}} = \frac{9}{12} = \frac{3}{4}$$

- 9 The equation of a cubic curve is  $y = 2x^3 - 9x^2 + 12x - 2$ .

(i) Find  $\frac{dy}{dx}$  and show that the tangent to the curve when  $x = 3$  passes through the point  $(-1, -41)$ . [5]

(ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]

(iii) Sketch the curve, given that the only real root of  $2x^3 - 9x^2 + 12x - 2 = 0$  is  $x = 0.2$  correct to 1 decimal place. [3]

i)

$$y = 2x^3 - 9x^2 + 12x - 2$$

$$\frac{dy}{dx} = 6x^2 - 18x + 12$$

$$\begin{aligned} \text{when } x = 3, \quad y &= 2(3)^3 - 9(3)^2 + 12(3) - 2 \\ &= 54 - 81 + 36 - 2 \\ &= 7 \end{aligned}$$

Point on curve  $(3, 7)$

$$\text{when } x = 3, \frac{dy}{dx} = 6(3)^2 - 18(3) + 12 \\ = 54 - 54 + 12 \\ = 12$$

Tangent at  $(3, 7)$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 12(x - 3)$$

$$y - 7 = 12x - 36$$

$$\underline{y = 12x - 29}$$

$$\text{when } x = -1, y = 12(-1) - 29 = -12 - 29 \\ = -41$$

$\therefore$  tangent at  $(3, 7)$  passes through  $(-1, -41)$

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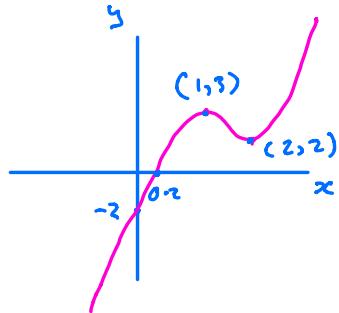
$$\text{i)} \text{ At t.p., } \frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0 \\ x^2 - 3x + 2 = 0 \\ (x-1)(x-2) = 0 \\ x = 1 \text{ or } x = 2$$

$$\text{when } x = 1, y = 2 - 9 + 12 - 2 \\ y = 3$$

$$\text{when } x = 2, y = 2(2)^3 - 9(2)^2 + 12(2) - 2 \\ = 16 - 36 + 24 - 2 \\ = 2$$

Turning points at  $(1, 3)$  and  $(2, 2)$

iii)



10

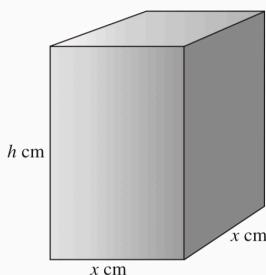


Fig. 10

Fig. 10 shows a solid cuboid with square base of side  $x$  cm and height  $h$  cm. Its volume is  $120 \text{ cm}^3$ .

(i) Find  $h$  in terms of  $x$ . Hence show that the surface area,  $A \text{ cm}^2$ , of the cuboid is given by  
$$A = 2x^2 + \frac{480}{x}$$
 [3]

(ii) Find  $\frac{dA}{dx}$  and  $\frac{d^2A}{dx^2}$ . [4]

(iii) Hence find the value of  $x$  which gives the minimum surface area. Find also the value of the surface area in this case. [5]

i)  $V = x^2 h = 120 \text{ cm}^3$

$$h = \frac{120}{x^2}$$

$$\begin{aligned} A &= 4xh + 2x^2 \\ &= 4x \times \frac{120}{x^2} + 2x^2 \\ &= \frac{480}{x} + 2x^2 \end{aligned}$$

$$\text{ii) } A = 480x^{-1} + 2x^2$$

$$\frac{dA}{dx} = -480x^{-2} + 4x = -\frac{480}{x^2} + 4x$$

$$\frac{d^2A}{dx^2} = 960x^{-3} + 4 = \frac{960}{x^3} + 4$$


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$$\text{iii) Min surface area when } \frac{dy}{dx} = 0$$

$$\Rightarrow -\frac{480}{x^2} + 4x = 0$$

$$-480 + 4x^3 = 0$$

$$4x^3 = 480$$

$$x^3 = 120$$

$$x = \sqrt[3]{120}$$

$$x = 4.9324$$

$$x = 4.93 \quad \text{to 3 s.f.}$$


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$$\text{Min Area} = \frac{480}{4.9324} + 2 \times 4.9324^2$$

$$= 145.97$$

$$= 146 \text{ cm}^2 \text{ to 3 s.f.}$$


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Check minimum when  $x = 4.93$

$$\frac{d^2y}{dx^2} = \frac{960}{4.93^3} + 4 > 0 \therefore \text{minimum}$$


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