Trigonometric Identities and Equations

$$
\tan \theta=\frac{\sin \theta}{\cos \theta} \quad \sin ^{2} \theta+\cos ^{2} \theta=1
$$

1) $\frac{\sqrt{1-\cos ^{2} \theta}}{\cos \theta \tan \theta}=\frac{\sqrt{\sin ^{2} \theta}}{\cos \theta \tan \theta}=\frac{\sin \theta}{\cos \theta \tan \theta}=\frac{\tan \theta}{\tan \theta}=1$
2) 

$$
\begin{aligned}
& 7 \sin ^{2} \theta+3 \cos \theta-5=0 \\
& 7\left(1-\cos ^{2} \theta\right)+3 \cos \theta-5=0 \\
& 7-7 \cos ^{2} \theta+3 \cos \theta-5=0 \\
& 7 \cos ^{2} \theta-3 \cos \theta-2=0
\end{aligned}
$$

By care $\cos \theta=0.7902$ or $\cos \theta=-0.3616$

$$
\cos ^{-1} 0.3616
$$

$$
=68.8^{\circ}
$$

$$
\begin{array}{c|c}
S & A \\
68 \cdot 2 & \\
\hline 6 \times 1 & \\
T & C
\end{array}
$$

$$
\theta=471.2^{\circ}
$$

4 i)

$$
\text { Show } \frac{1}{2} \sin ^{2} \theta+3-\frac{7}{2} \cos ^{2} \theta \equiv 4 \sin ^{2} \theta-\frac{1}{2}
$$

$$
\begin{aligned}
& \frac{1}{2} \sin ^{2} \theta+3-\frac{7}{2} \cos ^{2} \theta \\
= & \frac{1}{2} \sin ^{2} \theta+3-\frac{7}{2}\left(1-\sin ^{2} \theta\right) \\
= & \frac{1}{2} \sin ^{2} \theta+3-\frac{7}{2}+\frac{7}{2} \sin ^{2} \theta
\end{aligned}
$$

$$
\begin{aligned}
& \cos ^{-1} 0.7902 \\
& =37.8
\end{aligned}
$$

$$
\begin{aligned}
& \theta=397.8^{\circ}
\end{aligned}
$$

$$
=4 \sin ^{2} \theta-\frac{1}{2}
$$

ii) Solve $\frac{1}{2} \sin ^{2} \theta+3-\frac{7}{2} \cos \theta=0$

$$
\begin{aligned}
\Rightarrow \quad 4 \sin ^{2} \theta & =\frac{1}{2}
\end{aligned}=0 \quad \begin{aligned}
4 \sin ^{2} \theta & =\frac{1}{2} \\
\sin ^{2} \theta & =\frac{1}{8} \\
\sin \theta & = \pm \sqrt{\frac{1}{8}} \\
\sin \theta & =0.3536 \\
\sin ^{-1} 0.3536 & =20.7^{\circ}
\end{aligned}
$$



$$
\begin{aligned}
\theta=-200.7^{\circ},-159.3^{\circ}, & -20.7^{\circ} \\
& +20.7^{\circ}
\end{aligned}
$$

7) 

$$
\begin{aligned}
& \sin ^{3} \theta-2 \sin \theta+1=\cos ^{2} \theta \\
& \sin ^{3} \theta-2 \sin \theta+1=1-\sin ^{2} \theta \\
& \sin ^{3} \theta+\sin ^{2} \theta-2 \sin \theta=0 \\
& \sin \theta\left(\sin ^{2} \theta+\sin \theta-2\right)=0 \\
& \sin \theta(\sin \theta+2)(\sin \theta-1)=0
\end{aligned}
$$

$\Rightarrow \sin \theta=0$ or $\sin \theta=-2$ or $\sin \theta=1$

$$
\theta=0^{\circ}, 180^{\circ}, 360^{\circ}
$$

$$
\theta=90^{\circ}, 450^{\circ}
$$

