

Trigonometric Identities and Equations

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1) \quad \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta \tan \theta} = \frac{\sqrt{\sin^2 \theta}}{\cos \theta \tan \theta} = \frac{\sin \theta}{\cos \theta \tan \theta} = \frac{\tan \theta}{\tan \theta} = 1$$

$$3) \quad 7 \sin^2 \theta + 3 \cos \theta - 5 = 0$$

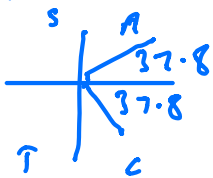
$$7(1 - \cos^2 \theta) + 3 \cos \theta - 5 = 0$$

$$7 - 7 \cos^2 \theta + 3 \cos \theta - 5 = 0$$

$$7 \cos^2 \theta - 3 \cos \theta - 2 = 0$$

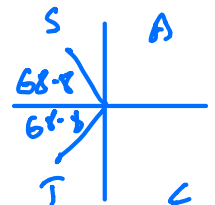
\Rightarrow calc $\cos \theta = 0.7902$ or $\cos \theta = -0.3616$

$$\cos^{-1} 0.7902 = 37.8$$



$$\underline{\theta = 397.8^\circ}$$

$$\cos^{-1} 0.3616 = 68.8^\circ$$



$$\underline{\theta = 471.2^\circ}$$

$$4) \quad \text{Show } \frac{1}{2} \sin^2 \theta + 3 - \frac{7}{2} \cos^2 \theta \equiv 4 \sin^2 \theta - \frac{1}{2}$$

$$\frac{1}{2} \sin^2 \theta + 3 - \frac{7}{2} \cos^2 \theta$$

$$= \frac{1}{2} \sin^2 \theta + 3 - \frac{7}{2} (1 - \sin^2 \theta)$$

$$= \frac{1}{2} \sin^2 \theta + 3 - \frac{7}{2} + \frac{7}{2} \sin^2 \theta$$

$$= 4 \sin^2 \theta - \frac{1}{2} \quad \checkmark$$

ii) Solve $\frac{1}{2} \sin^2 \theta + 3 - \frac{7}{2} \cos \theta = 0$

$$\Rightarrow 4 \sin^2 \theta - \frac{1}{2} = 0$$

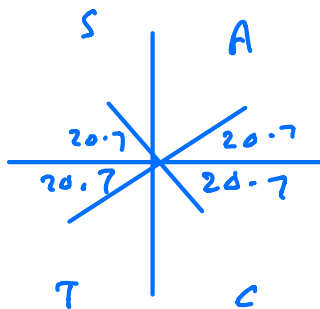
$$4 \sin^2 \theta = \frac{1}{2}$$

$$\sin^2 \theta = \frac{1}{8}$$

$$\sin \theta = \pm \sqrt{\frac{1}{8}}$$

$$\sin \theta = 0.3536$$

$$\sin^{-1} 0.3536 = 20.7^\circ$$



$$\theta = -200.7^\circ, -159.3^\circ, -20.7^\circ, +20.7^\circ$$

7) $\sin^3 \theta - 2 \sin \theta + 1 = \cos^2 \theta$

$$\sin^3 \theta - 2 \sin \theta + 1 = 1 - \sin^2 \theta$$

$$\sin^3 \theta + \sin^2 \theta - 2 \sin \theta = 0$$

$$\sin \theta (\sin^2 \theta + \sin \theta - 2) = 0$$

$$\sin \theta (\sin \theta + 2)(\sin \theta - 1) = 0$$

$$\Rightarrow \sin \theta = 0 \quad \text{or} \quad \cancel{\sin \theta = -2} \quad \text{or} \quad \sin \theta = 1$$

$$\theta = 0^\circ, 180^\circ, 360^\circ$$

$$\theta = 90^\circ, 450^\circ$$