

Algebraic Proofs GCSE Higher Tier A/A* Grades KS4 with Answers/Solutions

1. Prove that $(n + 4)^2 - (3n + 4) = (n + 1)(n + 4) + 8$
2. Prove that $(n + 4)^2 - (3n + 4) = (n + 2)(n + 3) + 6$
3. Prove that $(n + 3)^2 - (3n + 5) = (n + 1)(n + 2) + 2$
4. Prove that $(n - 5)^2 - (2n - 1) = (n - 3)(n - 9) - 1$
5. Prove that $(n - 3)^2 - (n - 5) = (n - 3)(n - 4) + 2$
6. Prove that $\frac{1}{2}(n + 1)(n + 2) - \frac{1}{2}n(n + 1) = n + 1$
7. Prove that $\frac{1}{4}(2n + 1)(n + 4) - \frac{1}{4}n(2n + 1) = 2n + 1$

8. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 6 for all positive integer values of n .
9. Prove that $(4n + 1)^2 - (4n - 1)^2$ is a multiple of 8 for all positive integer values of n .
10. Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5 for all positive integer values of n .



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10)

$$(5n+1)^2 - (5n-1)^2$$

$$25n^2 + 1 + 10n - [25n^2 + 1 - 10n]$$

~~$$25n^2 + 1 + 10n - 25n^2 - 1 + 10n$$~~

$$= 20n$$

$$= 5(4n) \quad \text{a multiple of 5}$$

9)

$$(4n+1)^2 - (4n-1)^2$$

$$= 16n^2 + 1 + 8n - [16n^2 + 1 - 8n]$$

$$\begin{aligned}
 &= 16n^2 + 1 + 8n - 16n^2 - 1 + 8n \\
 &= 16n \\
 &= 8(2n) \quad \text{a multiple of 8}
 \end{aligned}$$

15. Prove that $(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n .

16. Prove that $(n+1)^2 - (n-1)^2 + 4$ is always even for all positive integer values of n .

15) $(n+1)^2 - (n-1)^2 + 1$

$$\begin{aligned}
 &= n^2 + 1 + 2n - (n^2 + 1 - 2n) + 1 \\
 &= n^2 + 1 + 2n - n^2 - 1 + 2n + 1 \\
 &= 4n + 1 \\
 &= 2(2n) + 1 \quad \text{an odd number}
 \end{aligned}$$

16) $(n+1)^2 - (n-1)^2 + 4$

$$\begin{aligned}
 &= n^2 + 1 + 2n - (n^2 + 1 - 2n) + 4 \\
 &= n^2 + 1 + 2n - n^2 - 1 + 2n + 4
 \end{aligned}$$

$$= 4n + 4$$

$$= 2(2n+2) \text{ a multiple of 2}$$

and therefore even
